Was It Real? The Exchange Rate-Interest Differential Relation over the Modern Floating-Rate Period

RICHARD MEESDE and KENNETH ROGOFF*

ABSTRACT

In this paper, we explore the relationship between real exchange rates and real interest rate differentials in the United States, Germany, Japan, and the United Kingdom. Contrary to theories based on the joint hypothesis that domestic prices are sticky and monetary disturbances are predominant, we find little evidence of a stable relationship between real interest rates and real exchange rates. We consider both in-sample and out-of-sample tests. One hypothesis that is consistent with our findings is that real disturbances (such as productivity shocks) may be a major source of exchange rate volatility.

This paper investigates the empirical relationship between major currency real exchange rates and real interest rates over the modern (post-March 1973) flexible rate experience. The exchange rates examined here include the dollar/mark, dollar/yen, and dollar/pound rates. Our two major findings are as follows. First, the data do not indicate a strong correspondence between real interest rate differentials (short-term or long-term) and real exchange rates. This finding appears to conflict with the predictions of most monetary and portfolio balance models of exchange rate determination, though the conflict can be substantially reconciled if aggregate disturbances are primarily real in nature (i.e., changes in productivity, tastes, etc.). It is true that in many cases the sign of the estimated exchange rate-interest rate differential relationship is consistent with the possible predominance of financial market disturbances, but the relationship is not stable enough to be statistically significant. Second, although one does find some evidence of a unit root in both real exchange rates and long-term (but not short-term) real interest differentials, these two series do not appear to be linearly cointegrated. Hence, the nonstationarity (or near nonstationarity) in the two series cannot be attributed to the same factor.

In Section I, we briefly describe a class of small-scale monetary models of exchange rate determination. The importance of this class of models for empirical work derives from its strong predictions about how the exchange rate will move.

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in response to changes in a small number of fundamentals. We consider both in-
sample and rolling-regression out-of-sample tests. The methodology employed
here improves on earlier work in that we develop a test that allows us to compare
formally the out-of-sample fit of alternative models for multiperiod overlapping
forecasts. (See Appendix B.) Section II presents and evaluates the cointegration
results.

I. Quasi-Reduced-Form Real Exchange Rate Models

The models examined in this section are real versions of alternative rational-
expectations monetary models of exchange rate determination. Nominal versions
of these same models have been tested extensively. (See Meese and Rogoff [28,
29].) In the nominal rate models, the exchange rate depends on fundamentals
such as relative national money supplies, real incomes, short-term interest rates,
expected inflation differentials, and cumulated trade balances. In two earlier
studies [28, 29], we showed that these models forecast the nominal exchange rate
very poorly out of sample even when their forecasts are based on actual realized
values of the explanatory variables. This result is quite robust to a number of
plausible considerations.¹

A number of authors have posited that, despite the instability of nominal
exchange rates, there is nevertheless a strong relationship between real exchange
rates and real interest rates.² One rationale for this view is that, if the poor
performance of the nominal exchange rate regressions is primarily attributable
to money demand disturbances, there can still be a close correlation between real
interest differentials and real exchange rates. That is because, in the class of
monetary models considered here, unanticipated money demand disturbances
affect both variables proportionately.

The models we shall investigate are real versions of empirical models that have
been proposed by Dornbusch [6] and Frankel [13] (D-F) and by Hooper and
Morton [21] (H-M). These models are subsumed within a broader class of models
discussed in Obstfeld and Rogoff [34] and Meese and Singleton [30]. We refer
the reader to the latter two papers and to our earlier papers [28, 29] for theoretical

¹ In particular, the results are robust to a wide variety of estimation techniques, specifications of
the underlying money demand functions, alternative serial correlation or lagged adjustment corre-
cctions, alternative measures of forecast accuracy, different forecasting horizons, and out-of-sample
forecasting periods. (However, Woo [42] and Schinasi and Swami [37] have demonstrated that there
do exist particular sample periods for which a lagged adjustment specification or a random-coefficient
model appears to outperform the random-walk model.) Our two earlier studies covered the period
March 1973 through June 1981. We have updated our nominal exchange rate results through June
1984 using the methodology developed in Appendix B. The results were slightly more positive than
in our earlier studies in that several of the models did outperform the random-walk model, though
not significantly. (For the case of the dollar-pound rate, the results were borderline significant for
the Dornbusch-Frankel and Hooper-Morton models.) See our unabridged "Was It Real? The Ex-
details.

² See, for example, Feldstein [10] or the 1984 Economic Report of the President [3]. Shafer and
Loopesko [38] discuss theoretical models in which there is a relationship between real exchange rates
and real interest rates, and they also present some empirical evidence.
derivations. Underlying the D-F and H-M models is the assumption that goods market prices adjust slowly in response to anticipated disturbances and to excess demand. Consequently, less than perfectly anticipated monetary disturbances can cause temporary deviations in the real exchange rate from its long-run equilibrium value.

The interpretation of the empirical tests in this paper depends crucially on three assumptions, embodied in equations (1), (2), and (4) below. First (following D-F and H-M), we assume that any temporary deviation of the real exchange rate from its flexible-price equilibrium value is expected to damp out at a constant rate (in the absence of further shocks). That is, if the logarithm of the real exchange rate is \( q_t = s_t + p_t^* - p_t \), where \( s \) is the logarithm of the nominal exchange rate (domestic currency per foreign currency unit), \( p^* \) is the logarithm of the domestic-currency price of the domestically produced good, \( p^* \) is the logarithm of the foreign-currency price of the foreign good, and \( t \) is a time subscript, then

\[
E_t (q_{t+k} - \hat{q}_{t+k}) = \theta^k (q_t - \hat{q}_t), \quad 0 < \theta < 1, \tag{1}
\]

where \( E_t \) is the time-\( t \) expectations operator, \( \hat{q}_t \) is the real exchange rate that would prevail at time \( t \) if all prices were fully flexible, and \( \theta \) is the speed-of-adjustment parameter. \( \theta \) is a function of the structural parameters of the model. However, additive disturbances (such as money market shocks) do not affect \( \theta \). The monotonic adjustment property embodied in (1) is a feature of a fairly broad class of sticky-price rational-expectations monetary models, as is demonstrated by Obstfeld and Rogoff [34]. However, there are more general monetary and portfolio balance models in which monotonic adjustment need not hold. The resulting empirical models would, of course, be much less tractable without the monotonicity assumption.

In general, \( E_t (\hat{q}_{t+k}) \) will not equal \( \hat{q}_t \) unless there are no real shocks or unless all real shocks follow random-walk processes. A second crucial assumption (again following D-F and H-M) is that

\[
E_t \hat{q}_{t+k} = \hat{q}_t. \tag{2}
\]

In the H-M model, \( \hat{q}_t \) is actually posited to be a function of the cumulated current account (which itself is posited to follow a random walk). Substituting equation (2) into equation (1), one obtains

\[
q_t = \alpha (E_t q_{t+k} - q_t) + \hat{q}_t, \tag{3}
\]

where \( \alpha = 1/(\theta^k - 1) < -1 \).

The third important building block in the versions of the D-F and H-M models that we shall employ here is the uncovered interest-parity relation

\[
E_t s_{t+k} = s_t = \kappa r_t - \kappa R_t^*, \tag{4}
\]

where \( \kappa r_t \) is the \( k \)-period nominal interest rate at time \( t \).

Equation (4) implies that

\[
E_t (q_{t+k} - q_t) = \kappa R_t - \kappa R_t^*, \tag{5}
\]
where the $k$-period real interest rate

$$x R_e = x r_e - (E_r P_{t-k} - P_t).$$

Substituting equation (5) into equation (3) yields

$$q_t = \alpha (x R_e^* - x R^*) + \tilde{q}_t. \quad (6)$$

Relaxing (4) by allowing for an exogenous risk premium will add a forcing factor to (6). Equation (6) relates the real exchange rate to the real interest differential and to the flexible-price real exchange rate; this is the equation that will be used in our in-sample and out-of-sample tests. (In the empirical H-M model, $\hat{q}$ varies over time as a function of U.S. and foreign cumulated trade balances: $TB$ and $TB^*$, respectively. In the D-F model, it is constant.) Because of the monotonic-adjustment assumption (1), we can estimate equation (6) for real interest rates of any maturity. The maturity of the interest rate $k$ affects the coefficient $\alpha$ via the relation below equation (3). To check the robustness of our results to the monotonicity assumption, we consider results for both long- and short-term interest differentials.

A casual glance at real exchange data will indicate their near-unit-root properties. In order to determine whether or not to estimate equation (6) in levels or in first-difference form, we first consider some standard unit root tests. The results in Table I are based on regressions of the general form

$$q_t - q_{t-1} = b_0 + b_1 q_{t-1} + b_2 (q_{t-2} - q_{t-3}) + \epsilon_t. \quad (7)$$

The $b_i$ are constant parameters, and $\epsilon$ is a white-noise disturbance. If the autoregressive representation of $q_t$ contains a unit root (i.e., is integrated of order one), the $t$-ratio for $b_1$ should be consistent with the hypothesis $b_1 = 0$. Conventional $t$-tables are inappropriate for this hypothesis test, so we use the results of Dickey and Fuller [5] and the tabulated distribution in Fuller [14, p. 373] to interpret the $t$-ratio. The critical values for this ratio, using a five percent significance level for $T = 100$ and 250, are $-2.89$ and $-2.88$, respectively. From inspection of Table I, we cannot reject the null hypothesis of a unit root for the real exchange rate for any of the three exchange rates, when sampling is at

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**Table I**

Tests for Unit Roots in the Logarithm of the Real Exchange Rate Using Equation (7)*

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>$b_1$</th>
<th>$D^t$ t-Ratio for $b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar/Mark</td>
<td>-0.01</td>
<td>-0.7</td>
<td>-0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>Dollar/Pound</td>
<td>-0.03</td>
<td>-1.4</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>Dollar/Yen</td>
<td>-0.03</td>
<td>-1.5</td>
<td>0.15</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

*Monthly observations, February 1974 through March 1986. In all cases, no further lags of the dependent variable were necessary to whiten the residuals. All regressions included a constant and seasonal dummies.
Table II
Estimation of Equation (6) and Stability Test Using the GMM Estimator Described in Appendix A

<table>
<thead>
<tr>
<th>Real Exchange Rate*</th>
<th>Model</th>
<th>( \frac{R - R^*}{(\times 10)} )</th>
<th>Regressors* ( \times 10 )</th>
<th>Chi-Square Stability Test</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar/Mark</td>
<td>D-F</td>
<td>-0.917</td>
<td>269.8</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−0.388)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H-M</td>
<td>-0.439</td>
<td>6.478</td>
<td>5.115</td>
<td>96.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−1.172)</td>
<td>(0.438)</td>
<td>(1.362)</td>
<td></td>
</tr>
<tr>
<td>Dollar/Yen</td>
<td>D-F</td>
<td>-0.272</td>
<td>31.260</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−0.034)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H-M</td>
<td>-0.400</td>
<td>0.472</td>
<td>0.320</td>
<td>239.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−0.599)</td>
<td>(0.077)</td>
<td>(0.626)</td>
<td></td>
</tr>
<tr>
<td>Dollar/Pound</td>
<td>D-F</td>
<td>-0.088</td>
<td>11.170</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−0.651)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>H-M</td>
<td>-0.186</td>
<td>0.246</td>
<td>-1.631</td>
<td>690.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−0.509)</td>
<td>(0.226)</td>
<td>(−0.856)</td>
<td></td>
</tr>
</tbody>
</table>

* For the dollar/mark and dollar/pound rates, the range of the dependent variable is February 1974 through December 1985 (N = 147). For the dollar/yen rate, the range is February 1974 through January 1985 (N = 136).

b Instruments for the H-M model include the change in \( q_{t−1}, TB_{t−1}, TB^*_{t−1}, \) and \( (R \times 10)^{t−1} \). Instruments for the D-F model are the same except for the trade balances.

Numbers in parentheses are t-ratios. All regressors included seasonal dummies that are not reported to save space.

The trade balance variables are scaled as follows: U.S. \( TB \) in hundred billion dollars, the German \( TB^* \) in hundred billion marks, the U.K. \( TB^* \) in ten billion pounds, and the Japanese \( TB^* \) in ten trillion yen.

monthly intervals. Consequently, we will estimate the in-sample real exchange rate regressions in first-difference form.5

In-sample regressions based on equation (6) are presented in Table II. The estimates are based on monthly data for the period February 1974 through March 1986. We have chosen to annualize the variable \( R - R^* \); it is formed by taking either the annualized three-month or long-term nominal interest rate differential and subtracting off the annualized ex post realized three-month inflation differential as a proxy for its expected value (that is, by utilizing McCallum's [26] technique).4 The resulting equation is then estimated using an instrumental-variables GMM estimator. (See Appendix A.) The GMM estimator used here employs a heteroskedasticity- and autocorrelation-consistent covariance-matrix estimator.

5 Our results are robust to the possibility of conditional heteroskedasticity in the disturbance of (7), as Phillips [36]-type unit root tests lead to the same conclusions that are reported in the text.

4 This inflation proxy does not correspond to the term of the long interest rates, although both variables are measured in annual percentages. However, the results are best for the long-term rates (see below), so we report only these results in Table II.
The good news in Table II is that, for both models and for all three exchange rates, the real interest differential has the theoretically expected negative sign. A rise in the real interest differential in favor of the dollar leads to an appreciation of the real exchange rate (a fall in \( q \)). According to the estimates of the D-F model in Table II, if a money supply shock caused the annualized long-term interest rate differential to rise by one percentage point (in favor of dollar assets), the dollar would appreciate against the mark by \( \alpha = 0.92 \) percent in real terms.\(^5\)

The coefficients on the U.S. cumulated trade balance are of the wrong sign in all three cases, and the coefficients on the foreign cumulated trade balance are of the right sign except for the dollar/pound rate. The coefficients are insignificant in all cases, however.

The last column of Table II reports a chi-square test for a structural break in November 1980 (the Reagan election). The null hypothesis of no structural break is wildly rejected in all three cases.

The in-sample results reported in Table II are reinforced by the out-of-sample rolling-regression tests in Table III. Table III reports root mean square prediction error (RMSE) for the D-F and H-M models and for the random-walk model. (Comparisons involving mean absolute error (MAE) present a similar picture.) The results in Table III are also based on long-run real interest rate differentials. However, because using a GMM estimator in rolling regressions is extremely cumbersome, the results in Table III are based on Fair's [9] method. The expected inflation proxy used is the last twelve months' inflation instead of the realized inflation rate. Also, the regressions are estimated in level form with a correction for first-order serial correlation. (The results obtained using levels plus serial correlation correction were marginally better than those obtained using first differences.) As in our two earlier studies, the forecasts for the D-F and H-M models are formed using actual realized values of the real interest differential and the cumulated trade balances. Hence, Table III is viewed more accurately as measuring out-of-sample fit rather than forecasting ability.

The statistics in parentheses in Table III are for a test (see Appendix B) that involves formally comparing the forecasting performance of the D-F and H-M models with a random-walk model at multiple horizons. The two small-scale structural models do no better than the random-walk model for any forecast horizon (one, six, and twelve months) for the dollar-mark, dollar/yen, and dollar/pound exchange rates. Indeed, the random-walk model is significantly better in some cases.

To see whether the above results are peculiar to cross-rates against the dollar (perhaps because of U.S. productivity shocks, tax changes, or budget deficits),

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\(^5\) Assuming that the appropriate time interval for our model is monthly, the restriction on \( \alpha \) for the long-term interest differential (five- to ten-year government bond yields) is approximately \( \alpha < -0.0013 \) (for a 7½-year bond differential expressed in percentages). For three-month rates, the restriction would be \( \alpha < 0.04 \). The former restriction is satisfied for all long interest rate models except the D-F model for the dollar/pound exchange rate. Fewer of the short-term rate models satisfied the latter restriction and consequently are not reported. Finally, the implied speed of adjustment, \( \delta \), of the goods market is near one and hence quite slow. The result is consistent with other empirical work on real exchange rates; see, for example, Campbell and Clarida [2] or Hoizington [22].
we also consider statistics for the yen/mark, pound/mark, and pound/yen exchange rates. The H-M model does slightly improve on the random-walk model for the pound/yen rate, and, whereas the difference is small in magnitude, it is significant at one-month horizons.

We also considered a number of alternative model specifications without obtaining conspicuously different results. One specification involved using the short-term government bond yield differentials in place of long-term government bond yield differentials. Inflationary expectations were then proxied using the inflation rate over the preceding three months. Using short-term rates yielded no significant improvement. We also considered a lagged adjustment specification, obtained by adding $q_{t-1}$ as an additional explanatory variable in equation (6). Whereas some authors have reported improvements in similar regressions.

* These results are reported in more detail in an earlier working paper version of this study, cited in footnote 1. The results that are reported in the text but not presented in the tables are for a sample period that ends in March 1986.
using a lagged adjustment term (see footnote 1 above), the change yields no improvement here. Indeed, the lagged adjustment model performs very poorly at long horizons. We also tried to allow for coefficient drift in the rolling regressions by weighting observations by $0.95^{t-t_0}$, where $t_0$ is the most recent observation period used to form coefficient estimates. This modification did yield an improvement in some cases, but the improvement was never significant. Finally, we considered three purely statistical models as alternatives to the random-walk model: (1) a random walk with drift, (2) an ARIMA (0, 1, 1) or simple exponential smoothing, and (3) a local trend predictor. At monthly horizons, none of these time-series models forecast significantly better than the random-walk model.

What conclusions can we draw from the results reported in this section? Our evidence provides no support whatsoever for the view that a model (emphasizing the interaction of sticky prices and monetary disturbances) can explain the major swings in the real exchange rate. The strongest prediction of those models—that real interest differentials will be highly correlated with real exchange rate movements—simply does not appear in the data. Consequently, one must seriously consider the possibility that real shocks (such as the productivity disturbances emphasized in Kydland and Prescott’s [24] model) play a major role in buffeting exchange rates. Speculative bubbles are another possibility. For discussions of this issue, see Flood [12], Meese [27], Obstfeld and Rogoff [33, 35], Singleton [39], and West [40].

II. Tests of Cointegration of Real Exchange Rates and Real Interest Differentials

We have already shown in Table I that real exchange rates are sufficiently nonstationary that one cannot reject the hypothesis that there exists a unit root in the series. In this section, we show that the same is true for long-term real interest rate differentials, though not short-term differentials. We then use cointegration tests to explore the possibility that a single factor can explain the borderline nonstationarity in both series. The methods used in the preceding sections would not be powerful for detecting this type of relationship. If one were to find that the two series are cointegrated, it would provide evidence in favor of a structural model relating real exchange rates and real interest differentials. The concept of cointegration was first introduced by Granger [15] and later expanded upon by Engle and Granger [7]. A vector time series is cointegrated of order $(d, b)$ if each element needs to be differenced $d$ times to achieve stationarity but yet there exists a (not necessarily unique) linear combination of the two vectors that only needs to be differenced $(d-b)$ times to achieve stationarity. It is not unusual to find evidence of unit roots in the autoregressive representation of asset price data. In our context, we are interested in exploring the possibility

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Because of the marginal improvement in forecasting performance using weighted least squares and the pronounced evidence of parameter instability from our chi-square tests in Table II, we have run out-of-sample forecasting experiments with a rolling, fixed sample size of fifty observations. Such an exercise also obviates the need to make a bias correction to the statistics developed in Appendix B. Unfortunately, the fixed-sample, rolling-regression forecast statistics for the real exchange rate models are qualitatively unaffected by this modification.
that the nonstationarity we find in real exchange rates can be accounted for by
the nonstationarity of real interest differentials (or vice versa).

In Table IV, we report results of a test for a unit root in autoregressive
representations of the short- and long-term real interest differentials. The results
in Table IV are based on regressions of the same general form as equation (7).
Inspection of the table shows that we cannot reject the null hypothesis of a unit
root in the real long-term interest differential for all three currencies. However,
we reject it for the real short-term interest differential except for the dollar/yen
rate.

It is puzzling that real long-term interest differentials appear to be nonstation-
ary, given the current system of highly integrated capital markets. This anomaly
is also characteristic of the nominal long-term government bond rate differential,
at least for the data used in this study. As such, we cannot attribute the apparent
nonstationarity in the long-term interest differential to our expected inflation
proxy. (Inflation rates of industrialized countries tend to be stationary in any
case.) Both the nominal and real short-term interest differentials do appear to
be stationary in levels.8 The stationarity of short-term rates is consistent with
Mussa's [31] observation that forward premia exhibit much less volatility than
any of the individual variables related by covered interest parity. (Nominal
interest differentials equal the forward premium for regularly traded forward
rates.) The puzzling nonstationarity of long-term interest differentials might be
a consequence of the lack of homogeneity of our long-term bond yields, the use
of on-shore instead of Euromarket rates, or the lack of liquid forward markets
for long maturities.

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8 Test statistics for the hypothesis $b_i = 0$ in (7) for the U.S.-German, U.S.-Japanese, and U.S.-
U.K. three-month nominal interest rate differential are $-3.08$, $-1.82$, and $-3.54$, respectively. The
failure to reject the unit root for the U.S.-Japanese interest differential might be a consequence of
Japanese capital controls in effect over much of the sample period.

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Table IV
Tests for Unit Roots in the Short-Term and
Long-Term Real Interest Differentials Using
Equation (7)*

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>$\hat{b}_1$</th>
<th>$DF$ t-Ratio for $\hat{b}_1$</th>
<th>$\hat{b}_2$</th>
<th>$\hat{b}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dollar/Mark</td>
<td>$-0.17$</td>
<td>$-4.0^*$</td>
<td>$-0.22$</td>
<td>$-0.10$</td>
</tr>
<tr>
<td>Dollar/Pound</td>
<td>$-0.13$</td>
<td>$-3.2^*$</td>
<td>$0.03$</td>
<td>$0.11$</td>
</tr>
<tr>
<td>Dollar/Yen</td>
<td>$-0.05$</td>
<td>$-2.1$</td>
<td>$0.29$</td>
<td>$0.03$</td>
</tr>
<tr>
<td>Long-Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dollar/Mark</td>
<td>$-0.04$</td>
<td>$-2.0$</td>
<td>$-0.03$</td>
<td>$-0.02$</td>
</tr>
<tr>
<td>Dollar/Pound</td>
<td>$-0.02$</td>
<td>$-1.3$</td>
<td>$-0.08$</td>
<td>$-0.05$</td>
</tr>
<tr>
<td>Dollar/Yen</td>
<td>$-0.02$</td>
<td>$-1.3$</td>
<td>$0.05$</td>
<td>$-0.05$</td>
</tr>
</tbody>
</table>

* Monthly observations, February 1974 through March 1986. In
all cases, no further lags of the dependent variable were necessary
to whiten the residuals. *Denotes rejection of the unit root hypoth-
esis using at least a five percent significance level. All regressions
included a constant and seasonal dummies.
Another possible explanation of the nonstationarity of long-term real interest rate differentials is the low power of the unit root tests to detect borderline stationary alternatives. We believe, for example, that it is highly unlikely that real exchange rates have a unit root, but a coefficient of 0.95 in a regression of the real exchange rate on a single lag of itself is quite plausible. It suggests that deviations from purchasing power parity take many years to damp down. Given our sample size and a five percent significance level, the probability of a type-II error (accept a coefficient of 1.0 when it is really 0.95) is in the range of 0.8 to 0.8; see Evans and Savin [8, p. 771].

The nice feature of the cointegration tests reported below is that they can be meaningful even if there is high probability of a type-II error. Loosely speaking, they test whether some linear combination of the large variance components of real exchange rates and real interest differentials effectively cancel one another, leaving a residual with small variance. If there does not exist a linear combination of real exchange rates and real interest differentials that is itself a stationary process, it suggests that the relationship between the two variables is at best tenuous or that a highly variable factor has been omitted from the real exchange rate-real interest differential relation.

Since real short-term interest differentials appear to be stationary in levels, real exchange rate and real short-term interest differentials cannot be cointegrated. To test whether real exchange rates and real long-term interest rates are cointegrated, we employ the preferred tests of Engle and Granger [7]. A regression of $q$, on the real interest differential is run (or the reverse regression), and the residuals are examined for nonstationary behavior. The results in Table V suggest that real exchange rates and long-term real interest rate differentials are not cointegrated. Given this evidence, we should not necessarily expect to find damped (stationary) forecast errors when predicting real exchange rates with real interest rate differentials. Indeed, when the D-F and H-M models are estimated in levels form, the first-order serial correlation parameter is always close to unity.

Real interest differentials and real exchange rates are linked by international parity conditions, so our findings of no cointegration suggest that a variable omitted from relation, possibly the expected value of some future real exchange rate, must have large variance as well. Alternatively, the set of shocks inducing near nonstationarity in real exchange rates cannot be the same as the set of shocks impinging on real interest rate differentials.

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4 Huizinga [22] presents suggestive evidence that the long-run real exchange rate may have a mean-reverting component.

10 The forward regression (real exchange rate on the real interest differential) or the reverse regression can be used to test for cointegration. In the limit, the data matrix becomes colinear, so the coefficient of the reverse regression is equal to the reciprocal of the coefficient of the forward regression. Simultaneity does not pose a problem for the test procedure either; see Engle and Granger [7].

11 Isard [23] discusses the identity that links real exchange rates, real interest differentials, the expected future real exchange rate, and the risk premium.
Table V

Tests for Cointegration of Real Exchange Rates and Real Long-Term Interest Differentials

Tabulated results are based on the following regression:
\[ q_t = \text{constants} + \alpha (R_t - R_t^*) + \text{disturbance}. \]

Test 1: Reject the hypothesis of no cointegration at approximately a five percent significance level if the Durbin-Watson (DW) statistic exceeds 0.336; see Engle and Granger [7, Table II, p. 269].

Test 2: Reject the hypothesis of no cointegration at approximately a five percent significance level if the Dickey-Fuller (DF) t-ratio of equation (7) applied to the residuals of (i) is less than -3.17; see Engle and Granger [7, Table II, p. 269].

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\hat{\alpha}$</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar/Mark</td>
<td>-0.049</td>
<td>0.050</td>
<td>-1.320</td>
</tr>
<tr>
<td>Real Dollar/Pound</td>
<td>0.006</td>
<td>0.042</td>
<td>-1.454</td>
</tr>
<tr>
<td>Dollar/Yen</td>
<td>0.006</td>
<td>0.068</td>
<td>-1.188</td>
</tr>
</tbody>
</table>

* Mark and pound regressions are estimated over February 1974 through December 1985 (148 observations), and yen regressions over February 1974 through January 1985 (132 observations). All regressions include a constant and seasonal dummies.

III. Conclusions

The results we have presented are slightly more favorable than the results of our earlier studies. We do find that the real exchange rate and the real interest differential have the theoretically anticipated sign (although trade balance regressors tend not to have the anticipated sign). However, the relationship is not statistically significant, and real interest differentials do not provide significant improvement over a random-walk model in forecasting real exchange rates (except in a few isolated cases).

We have already alluded to one possible explanation of why monetary models perform so poorly, which is that the disturbances impinging on exchange markets are predominantly real. Thus, models that focus primarily on monetary disturbances should not be expected to explain very much. The "real shocks" hypothesis deserves further attention though it is not yet certain whether it will be helpful in building better empirical exchange rate models. It has proven extremely difficult to identify which real factors (such as technology shocks or changes in preferences) affected exchange rates over what periods. Still, it seems that further study along the lines of modern real business cycle research would be worthwhile. We have also mentioned another popular current explanation of the failure of monetary exchange rate models, which is the existence of self-fulfilling expectations or exchange market bubbles. However, Flood [12] has

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13 See, for example, Flood [10] or Barro [1]. Campbell and Clarida [2] also argue that real shocks may be important. Mark [28], however, finds evidence in favor of demand shocks over preference shocks.
demonstrated that the results of our rolling-regression methodology are robust to the possibility of (linear) rational bubbles. Finally, examination of the empirical implications of general equilibrium asset-pricing models of exchange rates, such as Hodrick [19], also merits further research.

Appendix A

The following GMM procedure was used to estimate the real exchange rate regressions in Table II. Let $q$ be the left-hand-side variable, $X$ the vector of right-hand-side variables, $\beta$ the coefficient matrix, and $u$ the disturbance term. Hence, the estimated regression is of the form:

$$ q = X\beta + u. \quad (A1) $$

Let $Z$ denote the instrument matrix, and use

$$ \hat{\beta}_m = (X'N_ZX)^{-1}X'N_Zq, \quad N_Z = Z(Z'Z)^{-1}Z' $$

as a preliminary estimate to secure the residuals $\bar{u} = y - X\hat{\beta}_m$. Then

$$ \hat{\beta}_{GMM(2L)} = (X'ZWZ'X)^{-1}(X'ZWZ'q), \quad (A3) $$

where $\hat{W}$ is the heteroskedasticity- and autocorrelation-consistent estimator of $[\lim_{T \to \infty} \left(\frac{1}{T}\right) E(Z'\mu\mu'Z)]^{-1}$. We form $\hat{W}$ using the Newey-West [32] formula for a positive-definite time-domain estimator:

$$ \hat{W} = \{\hat{\mu}_0 + \sum_{j=1}^{n-3} w(j, m)(\hat{\mu}_j + \hat{\mu}_j')\}^{-1}, \quad (A4) $$

where $w(j, m) = 1 - j/(m + 1)$ and $\hat{\mu}_j = \left(\frac{1}{T}\right) \sum_{t=j+1}^{T} \hat{\eta}_t \hat{\eta}_{t-j}$, where $\hat{\eta}_t$ has as its typical element $z_{it} \hat{\mu}_i$. Hence,

$$ \text{var}(\hat{\beta}_{GMM}) = (X'ZWZ'X)^{-1}, \quad (A5) $$

and the Wald Statistic for $R\beta = r$ (all dummies = 0) is thus

$$ (R\hat{\beta}_{GMM} - r)'[R \text{var}(\hat{\beta}_{GMM})R']^{-1}(R\hat{\beta}_{GMM} - r). $$

Dummies were used to test for the stability of $TB$, $TB^*$, $R - R^*$, and twelve seasonal intercept coefficients.

Appendix B

Let $e(1, t)$ and $e(2, t)$ denote the period-$t$ forecast errors from models (1) and (2), respectively. Let $A = [a_{ij}]$, $i, j = 1, 2$, denote the covariance matrix of forecast errors. Define $x(t) = e(1, t) - e(2, t)$ and $y(t) = e(1, t) + e(2, t)$. A test of the null hypothesis $a_{11} = a_{22}$ is easily conducted using $x(t)$ and $y(t)$, as $a_{11} = a_{22}$ only when $\text{cov}(x(t), y(t))$ is zero. Assuming that the vector process $(e(1, t), e(2, t))$ is independent and identically distributed as a $N(0, A)$, then a uniformly most powerful unbiased test of $a_{11} = a_{22}$ can be based on the sample correlation
coefficient of \(x(t)\) and \(y(t)\), \(\hat{\rho}(x, y)\). Specifically, a test of \(a_{11} = a_{22}\) can be based on a statistic with a Student's \(t\)-distribution with \(T - 2\) degrees of freedom:

\[
t_{T-2} = \hat{\rho}(x, y)(T - 2)^{1/2}/(1 - \hat{\rho}(x, y)^2)^{1/2},
\]

for \(T > 2\). (See Hogg and Craig [20, pp. 339–42].) \(T\) is the number of known forecast errors.

This procedure can only be applied to unbiased, serially uncorrelated, normally distributed forecast errors. For multiple \((k\text{-step-ahead})\) forecast horizons, a sequence of forecast errors will in general follow a moving average (MA) process of order \((k - 1)\). In this case, we can still use \(x(t)\) and \(y(t)\) to construct a test of \(a_{11} = a_{22}\), but asymptotic distribution theory for time series is required. Given our assumptions, \(x(t)\) and \(y(t)\) are themselves MA\((k - 1)\) stationary processes. Using results in Hannan [16, chapter 4], we know that the sample covariance of \(x(t)\) and \(y(t)\), \(\text{cov}(x(t), y(t))\), is a consistent estimator of the population covariance. In addition,

\[
\sqrt{T}[\text{cov}(x(t), y(t)) - \text{cov}(x(t), y(t))] \to N(0, B), \tag{B1}
\]

where \(B\) is a complicated function of the autocovariances and fourth cumulants of the vector process \((x(t), y(t))\); see Hannan [16, p. 209]. Assuming normality of forecast errors, we can ignore fourth cumulants. Exploiting the MA\((k - 1)\) behavior of optimal \(k\text{-step-ahead}\) forecasts, \(B\) can then be consistently estimated by

\[
\hat{B} = \sum_{t=-k+1}^{T-k-1} (1 - |s|/T)[\text{cov}(x(t), x(t - s))\text{cov}(y(t), y(t - s)) + \text{cov}(x(s), y(t - s))\text{cov}(y(s), x(t - s))]. \tag{B2}
\]

The test statistics reported in the paper are thus

\[
\text{cov}(x(t), y(t))/(\hat{B}/T)^{1/2}, \tag{B3}
\]

which is approximately \(N(0, 1)\) for large \(T\).

An alternative estimator of \(B\) that does not require normality of forecast errors can be calculated using the generalized method of moments (GMM) methodology described in Hansen [17], Hansen and Singleton [18], or White and Domowitz [41]. The estimate of the covariance of \(x(t)\) and \(y(t)\) is chosen to satisfy the orthogonality condition

\[
0 = \frac{1}{T} \sum [\text{cov}(x(t), y(t)) - (x(t) - \bar{x})(y(t) - \bar{y})]. \tag{B4}
\]

A consistent method-of-moments estimator of \(B\) is then

\[
\hat{B} = \sum_{t=-k+1}^{T-k-1} \frac{1}{T} \sum (1 - |s|/T)
\]

\[
x(t)y(t)x(t - s)y(t - s) - \text{cov}(x(t), y(t))^2. \tag{B5}
\]

The test statistic \(B(B)\) is again appropriate. The use of \((B2)\) or \((B5)\) to estimate \(B\) made no qualitative difference in the results reported in Table III.

Several caveats are in order. First, multiple step-ahead forecasts \((large \; k)\) are typically biased; the mean error of these forecasts (not reported in the text)
generally increases in absolute value with \( k \). Second, the assumption of normality of forecast errors is also suspect for large \( k \). (The normality assumption is not needed for the GMM methodology.) Third, our test statistic (B3) has not been subjected to any simulation experiments. Last, the rolling-regression estimates of \( e(1, t) \) and \( e(2, t) \) are based on different sample sizes. To correct for small-sample bias in (B3), the sample size can be kept constant across all regressions (see footnote 7) or forecast errors can be weighted by \( \sqrt{(T - t)/T} \), where \( T \) is the last (largest) sample and \( (T - t) \) is the number of observations in the regressions that generated \( e(1, t) \) and \( e(2, t) \).

**Appendix C**

The data are sampled monthly from February 1974 through March 1986. All the asset market data are end-of-month point sample from the Federal Reserve Board (FRB) data base. (In Meese and Rogoff [28, 29], asset market data were drawn from the same day as foreign money supply figures. The results do not seem to depend much on this issue, and end-of-month data are much more conveniently obtained.) The exchange rates are New York noon bids. The short-term interest rates are three-month interbank rates, and long-term rates are five- to ten-year government bond yields. The trade balance data are from the OECD, and CPI data are from the FRB. All raw series are seasonally unadjusted.

The Japanese-U.S. statistical results reported in the text utilize smaller data sets than the U.K.-U.S. and German-U.S. results, as we were missing conformable values of the Japanese long-term rate for the most recent observations.

**REFERENCES**

11. Robert P. Flood. "Explanations of Exchange Rate Volatility and Other Empirical Regularities in


