# Econometrics

Jerzy Mycielski

2010

# Topics

- *OLS* for conditional model
- Convergence in probability
- Limit theorems
  - Law of Large Numbers (LLN)
  - Central Limit Theorem (CLT)
- Cramer theorem
- Asymptotic properties of estimators
  - consistency
  - asymptotic normality
- Simultaneity

# Convergence in probability

- Convergence in probability is denoted as

$$a_n \xrightarrow{p} a$$

or

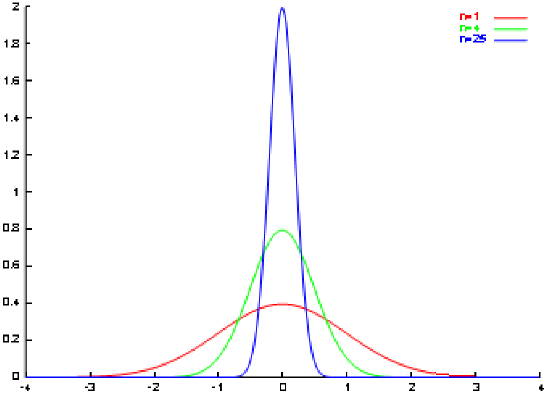
$$\text{plim}\left(a_n\right) = a$$

- Convergence in distribution is denoted as

$$a_n \xrightarrow{D} a$$

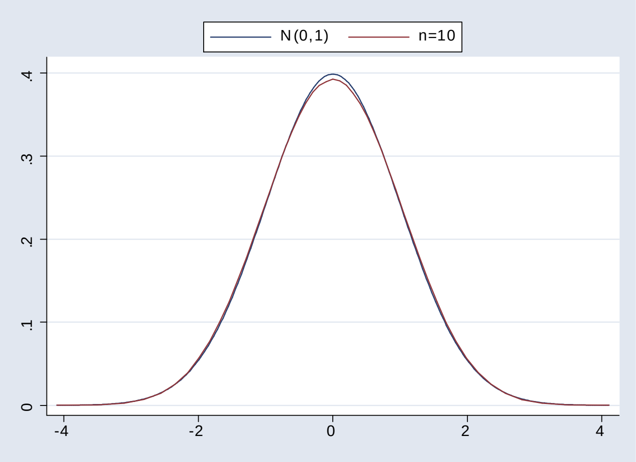
# Convergence in probability

Density of $\overline{x}$, where $x_i \sim N(0,1)$



Uniform distribution

Uniform distribution - density function of $\sqrt{n}\frac{\overline{x}-\mu}{\sigma^2}$



- If the assumption that

$$\mathrm{Cov}\left(\mathbf{x}_i, \varepsilon_i\right) = \mathrm{E}\left(\mathbf{x}_i' \varepsilon_i\right) = 0$$

  is invalid we say that in the model we have simultaneity problem
- In such a case OLS estimator is inconsistent
- This implies that in such a case even for vary large samples our estimates of $\boldsymbol{\beta}$ could be far away from parameters!

**Example**

(omitted variable) Variable $x_{3i}$ is omitted in the model:

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$$

and $\mathrm{Cov}\left(x_{2i}, x_{3i}\right) \neq 0$, so that in the model

$$y_i = \beta_1 + \beta_2 x_{2i} + \eta_i$$

where $\eta_i = \beta_3 x_{3i} + \varepsilon_i$ and covariance $\mathrm{Cov}\left(\eta_i, x_{2i}\right) \neq 0 \Longrightarrow$simultaneity

## Example

(feedback) In simplified Keynesian model

$$\begin{aligned} C_t &= a + bY_t + \varepsilon_t \\ Y_t &= C_t + I_t \end{aligned}$$

$C_t$ is consumption, $Y_t$ is *GDP* and $I_t$ investment.
Substituting for $C_t$ in the second equation

$$Y_t = a + bY_t + \varepsilon_t + I_t$$

Solving for $Y_t$:

$$Y_t = \frac{a}{1-b} + \frac{1}{1-b} I_t + \frac{\varepsilon_t}{1-b}$$

## Example (cont.)

Assume, that $\varepsilon_t$ and $I_t$ are not correlated, then

$$\begin{aligned} \mathrm{Cov}\left(\varepsilon_t, Y_t\right) &= \mathrm{Cov}\left(\varepsilon_t, \frac{a}{1-b} + \frac{1}{1-b} I_t + \frac{\varepsilon_t}{1-b}\right) \\ &= \frac{1}{1-b} \mathrm{Var}\left(\varepsilon_t\right) \neq 0 \end{aligned}$$

Simultaneity problem in this case is related to the fact that $C_t$ depends on $Y_t$ but at the same time $Y_t$ depends on $C_t$ (feedback)