Econometrics

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2009

Topics

1. Classical Linear Regression Model (CLRM)
   1. Assumptions of CLRM
2. Estimators - acceptibility criteria
   1. Expected value and unbiasedness of estimator **b** (necessary conditions)
   2. Variance and efficiency of estimator **b**: Gauss-Markow Theorem
3. Random errors and residuals
   1. Fundamental idempotent matrix
   2. Properties of fundamental idempotent matrix: idempentence, order, orthogonality to **X**

Topics

4. Estimator of the variance of the error term
   1. Expected value if the sum of squares
   2. Unbiasadness of s^2
   3. Degrees of freedom
5. Estimator of the linear combination of parameters and forecasting
   1. Unbiasadness of $\delta'\mathbf{b}$
   2. Variance of $\delta'\mathbf{b}$
   3. Estimator of element β_k
   4. Definiction of the forecast and error of forecast
   5. Variance of forecast and of ecomposition of variance of error of forecast

Basic properties of expected value, variance and trace

- Random variable $\mathbf{z}$
- Function $\mathbf{a} + \mathbf{B}\mathbf{z}$
- $\mathbf{a}$, $\mathbf{B}$ nonrandom

$$\mathrm{E}\left(\mathbf{a} + \mathbf{B}\mathbf{z}\right) = \mathbf{a} + \mathbf{B}\,\mathrm{E}\left(\mathbf{z}\right)$$

$$\mathrm{Var}\left(\mathbf{a} + \mathbf{B}\mathbf{z}\right) = \mathbf{B}\,\mathrm{Var}\left(\mathbf{z}\right)\mathbf{B}'$$

- Properties of trace:
  - if $\mathbf{AB}$ i $\mathbf{BA}$ exist, then

$$\begin{aligned} \mathrm{tr}\left(\mathbf{A} + \mathbf{B}\right) &= \mathrm{tr}\left(\mathbf{A}\right) + \mathrm{tr}\left(\mathbf{B}\right) \\ \mathrm{tr}\left(\mathbf{AB}\right) &= \mathrm{tr}\left(\mathbf{BA}\right) \end{aligned}$$

  - for random matrix $\mathbf{A}$

$$\mathrm{E}\left[\mathrm{tr}\left(\mathbf{A}\right)\right] = \mathrm{tr}\left[\mathrm{E}\left(\mathbf{A}\right)\right]$$

Estimated parameters for Cobba-Douglasa production function (years 1995.4 - 2005.2)

Source	SS	df	MS
Model	.246551725	3	.082183908
Residual	.104934571	35	.002998131
Total	.351486297	38	.009249639

Number of obs	= 39
F(3, 35)	= 27.41
Prob > F	= 0.0000
R-squared	= 0.7015
Adj R-squared	= 0.6759
Root MSE	= .05476

q	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
l	1.236623	.3801312	3.25	0.003	.4649154	2.00833
r	.0080266	.017226	0.47	0.644	-.026944	.0429972
t	.0106685	.0014495	7.36	0.000	.0077258	.0136112
_cons	-5.31012	3.678503	-1.44	0.158	-12.77788	2.157639

- $s^2 = \frac{RSS}{N-K} = \frac{.104934571}{39-4} = 0.0029981$ (Mean Squared Error)
- $s = \sqrt{s^2} = \sqrt{0.0029981} = 0.05476$ (Root of **M**ean **S**quared **E**rror)

Estimated variance and covariance matrix

```
             |        l         r         t     _cons
-------------+----------------------------------------
           l |    .1445
           r | -.001274   .000297
           t |   .00046 -1.1e-06   2.1e-06
       _cons | -1.39819   .013099 -.004455   13.5314
```

Linear combination of parameters

- Wage model with interactions between gender and education

$$\log(\text{wage}) = \beta_0 + \beta_1 \text{secondary} + \beta_2 \text{higher} + \beta_3 \text{sex} + \\ + \beta_4 \text{secondary} \times \text{sex} + \beta_5 \text{higher} \times \text{sex} + \beta_6 \text{age} + \varepsilon$$

- The wage differential between primary and higher education for females $\beta_2 + \beta_5$
- Estimator $b_2 + b_5$

Regression results

lplaca	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
_Ieduc_3_1	.6518608	.0206535	31.56	0.000	.6113731	.6923485
_Ieduc_3_2	.1766364	.0180894	9.76	0.000	.1411753	.2120976
_Isex_2	-.3560174	.0198582	-17.93	0.000	-.394946	-.3170887
_IeduXse~1_2	.0747255	.0302043	2.47	0.013	.0155151	.133936
_IeduXse~2_2	.1710516	.0270627	6.32	0.000	.1179998	.2241034
age	.0125822	.0005636	22.33	0.000	.0114775	.013687
_cons	6.966938	.0254847	273.38	0.000	6.91698	7.016897

- $b_2 + b_5 = 0.652 + 0.074 = 0.726$

Estimator of variance and standard error of single parameter

- Cobba-Douglasa Model

$$q_t = \beta_0 + \beta_1 l_t + \beta_2 r_t + \beta_3 t + \varepsilon_t$$

- Estimator of β_1 is equal to $b_1 = 1.236623$
- Estimator of $\mathrm{Var}(b_1) = .1445$
- Estimator of $se(b_1) = \sqrt{.1445} = .38013$
- Estimated model representation

$$q = \underset{(3.679)}{-5.310} + \underset{(0.380)}{1.237} \times l + \underset{(0.017)}{0.008} \times r + \underset{(0.001)}{0.011} \times t$$

Forecast

- Philips curve model

$$\text{unemployment}_t = 19.663 - 0.411 \times \text{gdp}_t - 0.232 \times \text{infl}_t$$

- Forecast of unemployment for growth of gdp 4% i inflation 3%. Exogenous variable vector

$$\mathbf{x}_f = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix}.$$

$$\text{unemployment}_f = 19.663 - 0.411 \times 4 - 0.232 \times 3 = 17.323$$

- Variance of forecast

$$
\begin{aligned}
\mathbf{x}_f \; {}^{\circ}{}_{\mathbf{b}}\mathbf{x}'_f &= \begin{bmatrix} 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} .695497 & -.120855 & -.003656 \\ -.120855 & 0.0426 & -0.0061 \\ -.003656 & -0.0061 & 0.0029 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \\
&= .268
\end{aligned}
$$

- Estimated standard daviation of random error $s^2 = 6.1$
- Variance of forecast error: $.268 + 6.1 = 6.368$
- Standard error of forecast error $\sqrt{6.368} = 2.523$