# Three stage Least Squares ($3SLS$)

- Variance matrix of $\boldsymbol{u}_t$ is given by$\mathrm{Var}\left(\boldsymbol{u}_t\right) = \boldsymbol{\Sigma}$

- Its elements can be interpreted as variances and covariances between simultaneous error terms in equations of the structural form.

- Variance matrix of the error term for equation $i$ and time $1, \ldots, T$ is equal to (homoscedasticity, no autocorrelation):

$$\mathrm{Var}\left(\boldsymbol{u}_i\right) = \boldsymbol{\sigma}_{ii}\boldsymbol{I}$$

- Covariance between error terms in equations $i$ and $j$

$$\mathrm{Cov}\left(\boldsymbol{u}_i, \boldsymbol{u}_j\right) = \boldsymbol{\sigma}_{ij}\boldsymbol{I}$$

- Then the error term for stacked model $\overline{\boldsymbol{u}}$ is heteroscedastic and the variance of this error term is given by:

$$\mathrm{Var}\left(\overline{\boldsymbol{u}}\right) = \begin{bmatrix} \boldsymbol{\sigma}_{11}\boldsymbol{I} & & \boldsymbol{\sigma}_{1G}\boldsymbol{I} \\ & \ddots & \\ \boldsymbol{\sigma}_{G1}\boldsymbol{I} & & \boldsymbol{\sigma}_{GG}\boldsymbol{I} \end{bmatrix} = \overline{\boldsymbol{\Sigma}}$$

where $\boldsymbol{\sigma}_{ij}$ is equal to $i, j$ element of $\boldsymbol{\Sigma}$.

- If we have a consistent estimator $\overline{\boldsymbol{\beta}}$ is possible to estimate elements of $\boldsymbol{\Sigma}$ as empirical covariances between residuals from equations of the structural form.

$$\widehat{\boldsymbol{\Sigma}}_{ij} = \widetilde{\boldsymbol{\sigma}}_{ij} = \frac{\widetilde{\boldsymbol{e}}_i'\widetilde{\boldsymbol{e}}_j}{T}$$

- But $\overline{\boldsymbol{\Sigma}}$ is just a simple transformation of $\boldsymbol{\Sigma}$.

- Then it is possible to obtain more efficient estimator by using the Generalized Least Squares ($GLS$) rather then $OLS$ for $\widehat{\overline{\overline{\boldsymbol{Z}}}}$.

- $3SLS$ method

  1. same as in $2SLS$
  2. same as in $2SLS$
  3. from estimated structural form obtain the residuals calculate $\widetilde{\boldsymbol{\sigma}}_{ij} = \frac{\widetilde{\boldsymbol{e}}_i' \widetilde{\boldsymbol{e}}_j}{T}$, form

$$\widehat{\overline{\overline{\boldsymbol{\Sigma}}}} = \begin{bmatrix} \widetilde{\boldsymbol{\sigma}}_{11}\boldsymbol{I} & & \widetilde{\boldsymbol{\sigma}}_{1G}\boldsymbol{I} \\ & \ddots & \\ \widetilde{\boldsymbol{\sigma}}_{G1}\boldsymbol{I} & & \widetilde{\boldsymbol{\sigma}}_{GG}\boldsymbol{I} \end{bmatrix}$$

  and estimate the parameter vector with $GLS$ estimator

$$\boldsymbol{b}_{3SLS} = \left( \widehat{\overline{\overline{\boldsymbol{Z}}}}' \widehat{\overline{\overline{\boldsymbol{\Sigma}}}}^{-1} \widehat{\overline{\overline{\boldsymbol{Z}}}} \right)^{-1} \widehat{\overline{\overline{\boldsymbol{Z}}}}' \widehat{\overline{\overline{\boldsymbol{\Sigma}}}}^{-1} \overline{\boldsymbol{y}}$$

- $3SLS$ is called full information method as it requires the specification of all the equations of the model

- Second most popular full information method of estimation of simultaneous equation models is Full Information Maximum Likelihood ($FIML$)

**Example 1.** *(Klein model) One of the first structural models estimated was model of the US economy in years* $1921-1941$ *estimated by Klein. It consists of the following equations:*

$$
\begin{array}{ll}
C_t = \boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2 P_t + \boldsymbol{\alpha}_3 P_{t-1} + \boldsymbol{\alpha}_4 \left(W_t + W_t'\right) + u_{1t} & \textit{consumption} \\
W_t = \boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 X_t + \boldsymbol{\gamma}_3 X_{t-1} + \boldsymbol{\gamma}_4 t + u_{3t} & \textit{wages in private sector} \\
I_t = \boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 P_t + \boldsymbol{\beta}_3 P_{t-1} + \boldsymbol{\beta}_4 K_{t-1} + u_{2t} & \textit{investments} \\
K_t = I_t + K_{t-1} & \textit{capital} \\
X_t = C_t + I_t + G_t & \textit{GDP identity} \\
P_t = X_t - W_t - T_t & \textit{net profits in private sector}
\end{array}
$$

*Endogenous variables in this model are consumption* $C_t,$ *wages in private* *sector $W_t$, investments $I_t$, capital $K_t$, GDP $X_t$ and net profits in private sector $P_t$. Exogenous variables are the following: constant, linear trend $t$, government expenditure $G_t$, wages in public sector $W_t'$, taxes $T_t$ and lagged variables $P_{t-1}$, $K_{t-1}$, $X_{t-1}$. Profits and wages are separated in consumption equation as the propensity to spent from income from wages is different than to spend income from profits (at least at aggregate level). In this model we have no monetary sector - this is the main difference with standard IS/LM model.*

- Estymation of one equation with $IV$

```
Instrumental variables (2SLS) regression

      Source |       SS       df       MS              Number of obs =      21
-------------+------------------------------           F(  3,    17) =  225.93
       Model |  919.504138     3  306.501379           Prob > F      =  0.0000
    Residual |  21.9252518    17  1.28972069           R-squared     =  0.9767
-------------+------------------------------           Adj R-squared =  0.9726
       Total |  941.429389    20  47.0714695           Root MSE      =  1.1357

------------------------------------------------------------------------------
c            |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
p            |   .0173022   .1312046     0.13   0.897    -.2595153    .2941197
wp_plus_wg   |   .8101827   .0447351    18.11   0.000        .7158    .9045654
p            |
          L1 |   .2162338   .1192217     1.81   0.087    -.0353019    .4677696
_cons        |   16.55476   1.467979    11.28   0.000     13.45759    19.65192
------------------------------------------------------------------------------
Instrumented:  p L.p wp_plus_wg
Instruments:   year g wg t L.p k1 L.x
------------------------------------------------------------------------------
```

- Sargan test for overidentifing restrictions

```
Tests of overidentifying restrictions:
Sargan N*R-sq test          8.772  Chi-sq(4)    P-value = 0.0671
Basmann test                9.325  Chi-sq(4)    P-value = 0.0535
```

- Hausaman-Wu test for edogeneity of explanatory variables

```
Tests of endogeneity of: p L.p wp_plus_wg
H0: Regressors are exogenous
    Wu-Hausman F test:                    3.48647  F(3,14)     P-value = 0.04460
    Durbin-Wu-Hausman chi-sq test:        8.98009  Chi-sq(3)   P-value = 0.02956
```

- $2SLS$ all equations

```
------------------------------------------------------------------------------
             |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
c            |
p            |
          L1 |   .2162338   .1192217     1.81   0.076    -.0231137    .4555814
          -- |   .0173022   .1312046     0.13   0.896     -.246102    .2807064
wp_plus_wg   |   .8101827   .0447351    18.11   0.000     .7203733    .8999921
_cons        |   16.55476   1.467979    11.28   0.000     13.60767    19.50185
-------------+----------------------------------------------------------------
```

```
wp           |
x            |
          -- |   .4388591   .0396026    11.08   0.000     .3593535    .5183646
          L1 |   .1466739   .0431639     3.40   0.001     .0600187     .233329
year         |   .1303956   .0323884     4.03   0.000     .0653733     .195418
_cons        |  -250.2936   61.95692    -4.04   0.000    -374.6773   -125.9099
-------------+----------------------------------------------------------------
i            |
p            |
          -- |   .1502219   .1925335     0.78   0.439    -.2363053    .5367491
          L1 |   .6159434   .1809258     3.40   0.001     .2527198    .9791671
k1           |  -.1577876   .0401521    -3.93   0.000    -.2383963    -.077179
_cons        |   20.27821   8.383247     2.42   0.019     3.448138    37.10828
------------------------------------------------------------------------------
Endogenous variables:  c wp i wp_plus_wg x p
Exogenous variables:   L.p L.x year k1 g wg t
------------------------------------------------------------------------------
```

- $3SLS$ all equations

| | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| c | | | | | | |
| p | | | | | | |
| L1 | .1631439 | .1004382 | 1.62 | 0.104 | -.0337113 | .3599992 |
| -- | .1248904 | .1081291 | 1.16 | 0.248 | -.0870387 | .3368194 |
| wp_plus_wg | .790081 | .0379379 | 20.83 | 0.000 | .715724 | .8644379 |
| _cons | 16.44079 | 1.304549 | 12.60 | 0.000 | 13.88392 | 18.99766 |
| wp | | | | | | |
| x | | | | | | |
| -- | .4004919 | .0318134 | 12.59 | 0.000 | .3381388 | .462845 |
| L1 | .181291 | .0341588 | 5.31 | 0.000 | .1143411 | .2482409 |
| year | .149674 | .0279352 | 5.36 | 0.000 | .094922 | .2044261 |
| _cons | -287.2233 | 53.4488 | -5.37 | 0.000 | -391.9811 | -182.4656 |
| i | | | | | | |
| p | | | | | | |
| -- | -.0130791 | .1618962 | -0.08 | 0.936 | -.3303898 | .3042316 |
| L1 | .7557238 | .1529331 | 4.94 | 0.000 | .4559805 | 1.055467 |
| k1 | -.1948482 | .0325307 | -5.99 | 0.000 | -.2586072 | -.1310893 |
| _cons | 28.17785 | 6.793768 | 4.15 | 0.000 | 14.86231 | 41.49339 |

```
Endogenous variables:  c wp i wp_plus_wg x p
Exogenous variables:   L.p L.x year k1 g wg t
------------------------------------------------------------------------------
```

## Lukas critique of classical econometrics

- Equations of structural model describe the behavior of rational agents
- In classical structural model behavior of agents was explained by changes of exogenous variables
- However: the form of structural equations were often not based on the principle of agents rationality (were lacking microfoundations)
- But: if the process generating the exogenous variables changes the rational agent will take in account and will adjust his behavior to maximize his objective function
- As a result the parameters of the model may change

- The most important conclusion is that predictions about the results of the changes of the government policy based on classical structural model can prove to be completely misleading

**Example 2.** *Demand for labor*

$$
\begin{array}{ll}
l = -\boldsymbol{\beta}_0 + gdp - \boldsymbol{\beta}_1 t + \boldsymbol{\varepsilon}_1 & \textit{demand for labour (in hours)} \\
x = l - h & \textit{demand for labour (in number of workers)}
\end{array}
$$

*where $l$ is the log of total number of hours worked, $gdp$ is the log of gross domestic product, $h$ is the log of the average number of hours worked and $x$ is the number of workers. $\exp(\boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 t)$ can be interpreted as log of unit labor cost and $\boldsymbol{\beta}_1$ is the average growth rate of unit labor costs. $gdp, h$ are exogenous in this model. Parameters $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_1$ has to be estimated.*

*According to this model government can increase employment changing $h$. It can be done by reducing working hours.*

*France tried to reduce unemployment in this way - result was either nil or negative. It seems that neither* $\boldsymbol{\beta}_0, \boldsymbol{\beta}_1$ *is constant nor* $gdp$ *is exogenous as* $h$ *changes.*

*Solution: to predict the result of the change of the government policy with respect to working hours we should build the equilibrium model in which agents are rational, derive the structural model an the base of this economic model and estimate it parameters.*

- Conclusion:
  - equations of structural model should be based on microfoundations

# Sims critique of classical econometrics

- Forecast from large simultaneous equations models are not better the forecasts from atheoretical $ARIMA$ models
- High $R^2$ and significant $t$ statistics where often obtained by unstructured data mining
- Sims critique
  - Economic theory is mostly related to equilibrium conditions
  - Therefore in structural dynamic dynamic models the form of structural equations cannot be based in theory
  - It is impossible do distinguish between exogenous and endogenous variables - everything depend on everything

- Sims solution to the problems of structural modelling:
  - Analyze the dynamic features of the model rather than its structure
  - It is especially important to identify the reactions of the model to the shocks (government interventions, unexpected actions of central bank etc.)