Estimation of dynamic panel models

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First difference estimator

- Necessary assumptions for consistency of estimator

$$\mathrm{E}\left(u_{it} \middle| \boldsymbol{x}_i, c_i \right) = 0 \text{ for } t = 1, \ldots, T$$

- We use differencing to eliminate individual effect c_i
- Model

$$y_{it} = \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it}$$

- We apply first order differences and obtain

$$\Delta y_{it} = \Delta \boldsymbol{x}_{it}\boldsymbol{\beta} + \Delta u_{it}$$

- First observation $(T = 1,\ i = 1, \ldots, N)$ is omitted
- Notice that the variables with no time variation are also omitted
- First difference estimator (FD) we obtain by regressing on pooled sample $(POLS)$ Δy_{it} on $\Delta \boldsymbol{x}_{it}$
- This estimator is consistent as, for the assumption made $\mathrm{E}\left(\Delta \boldsymbol{x}_{it}' \Delta u_{it}\right) = 0$
- Under assumptions, that $\mathrm{E}\left(\Delta \boldsymbol{u}_i \Delta \boldsymbol{u}_i' \middle| \boldsymbol{x}_i, \boldsymbol{c}_i \right) = \sigma_u^2 \boldsymbol{I}_T$ this estimator is also the efficient

Panel estimators under sequential moment restrictions

- Typical assumptions necessary for consistency of *RE*, *FE*, *FD* are based on strict exogeneity:

$$\mathrm{E}\left(u_{it} \middle| \boldsymbol{x}_{i1}, \ldots, \boldsymbol{x}_{iT} \right)$$

- This assumption implies that the expected value of idiosyncratic error (u_{it}) does not depend (is not correlated) on past contemporaneous and *future* values of explanatory variables $\boldsymbol{x}_i$
- Consider following model:

$$y_{it} = \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it}, \quad \text{for } t = 1, 2, \ldots, T, \; i = 1, \ldots, N$$

  and assume, that expected value of u_{it} may depend on *future* values of $\boldsymbol{x}_{it}$ but not on the past and contemporaneous values of $\boldsymbol{x}_{it}$
- So we assume the following *sequential moment restrictions*

$$\mathrm{E}\left(u_{it} \middle| \boldsymbol{x}_{it}, \boldsymbol{x}_{it-1}, \ldots, \boldsymbol{x}_{i1}, c_i \right) = 0, \; i = 1, \ldots, N$$

- This restrictions imply that expected value of u_{it} cannot depend on contemporaneous and past values of $\boldsymbol{x}_i$
- If this assumptions are true we say that $\boldsymbol{x}_{it}$ is sequentially exogenous with respect to individual effect
- Linear unobserved effect model can be formulated now as follows:

$$\mathrm{E}\left(y_{it}\middle|\,\boldsymbol{x}_{it},\boldsymbol{x}_{it-1},\ldots,\boldsymbol{x}_{i1},c_i\right)=\mathrm{E}\left(y_{it}\middle|\,\boldsymbol{x}_{it},c_i\right)=\boldsymbol{x}_{it}\boldsymbol{\beta}+c_i$$

Przykład: *(Wooldrodge) Dynamic unobserved effect model*

$$y_{it}=\boldsymbol{z}_{it}\gamma+\rho_1 y_{i,t-1}+c_i+u_{it}$$

so in this case$x_{it}\equiv\left(\boldsymbol{z}_{it},y_{i,t-1}\right)$. *Therefore* $\left(\boldsymbol{x}_{it},\boldsymbol{x}_{it-1},\ldots,\boldsymbol{x}_{i1}\right)=\left(\boldsymbol{z}_{it},y_{i,t-1},\ldots,\boldsymbol{z}_{i1},y_{i,0}\right)$ *and sequential model restrictions imply, that*

$$\begin{aligned}\mathrm{E}\left(y_{it}\middle|\,\boldsymbol{z}_{it},y_{i,t-1},\ldots,\boldsymbol{z}_{i1},y_{i,0},c_i\right)&=\mathrm{E}\left(y_{it}\middle|\,\boldsymbol{z}_{it},y_{i,t-1},c_i\right)\\&=\boldsymbol{z}_{it}\gamma+\rho_1 y_{i,t-1}+c_i\end{aligned}$$

Sequential exogeneity and consistency of panel estimators

- In the case when sequential moment conditions are valid but the assumption of strict exogeneity is false *RE*, *FE*, *FD* estimators are not consistent. For instance fixed effect estimator is not consistent as:

$$\text{plim}\left(\widehat{\beta}_{FE}\right) = \beta + \left[T^{-1}\sum_{t=1}^{T}\mathrm{E}\left(\ddot{\boldsymbol{x}}_{it}\ddot{\boldsymbol{x}}_{it}'\right)\right]^{-1}\left[T^{-1}\sum_{i=1}^{T}\mathrm{E}\left(\ddot{\boldsymbol{x}}_{it}'u_{it}\right)\right]$$

  but $\mathrm{E}\left(\ddot{\boldsymbol{x}}_{it}'u_{it}\right) = \mathrm{E}\left[\left(\boldsymbol{x}_{it} - \overline{\boldsymbol{x}}_{it}\right)u_{it}\right] = -\mathrm{E}\left(\overline{\boldsymbol{x}}_{it}u_{it}\right) = T^{-1}\sum_{s=1}^{T}\mathrm{E}\left(\boldsymbol{x}_{is}u_{it}\right) = T^{-1}\sum_{s=t+1}^{T}\mathrm{E}\left(\boldsymbol{x}_{is}u_{it}\right) \neq 0$, because in the case of sequential moment conditions we do not assume that correlation between u_{it} and $\boldsymbol{x}_{is}$ is zero for $s > t$.
- The asymptotic bias in this case is vanishing with rate T^{-1} but for panels T is usually small

Sequential exogeneity and consistency of panel estimators cont.

- If $\boldsymbol{x}_{it}$ is stationary then *FE* estimator is better than *FD* estimator as the estimator *FE* has bias $O\left(T^{-1}\right)$ and for *FD* estimator bias does not depend on T
- However it is possible to find consistent estimators under sequential moment conditions
- Applying first differences we obtain

$$\Delta y_{it} = \Delta \boldsymbol{x}_{it}\boldsymbol{\beta} + \Delta u_{it}, \quad \text{for } t = 2, \ldots, T$$

- Sequential moment exogeneity of $\boldsymbol{x}_i$ imply that

$$\mathrm{E}\left(\boldsymbol{x}'_{is} u_{it}\right) = 0, \quad \text{for } s = 1, 2, \ldots, t$$

- Notice however, that

$$\mathrm{E}\left(\Delta \boldsymbol{x}'_{it} \Delta u_{it}\right) \neq 0$$

as according to assumptions made $\boldsymbol{x}_{it}$ can be correlated with u_{it-1}. Therefore *POLS* estimator for the model on first differences is not consistent

Consistent estimator for the differenced model with sequentially exogenous variables

- Notice that

$$\mathrm{E}\left(\boldsymbol{x}_{is}'\Delta u_{it}\right) = 0, \qquad \text{for } s = 1, 2, \ldots, t-1$$

  so that

$$\mathrm{E}\left(\Delta\boldsymbol{x}_{is}'\Delta u_{it}\right) = 0, \qquad \text{for } s = 1, 2, \ldots, t-1$$

- We conclude that vectors of variables $\boldsymbol{x}_{it}^{o} = \boldsymbol{x}_{it-1}, \ldots, \boldsymbol{x}_{i,0}$ or $\boldsymbol{x}_{it}^{o} = \Delta\boldsymbol{x}_{it-1}, \ldots, \Delta\boldsymbol{x}_{i,0}$ (or any linear combination or function of them) can be used as instrumental variables in the equation on first differences
- In effect first differenced equation can be estimated with $2SLS$ applied to the model on first differences and estimated on pooled sample

Efficient estimator for the model with sequentially exogenous variables

- Usually assumption necessary condition for application of $2SLS$ (correlation of instruments with explanatory variables) which is in this case $\mathrm{E}\left(\Delta \boldsymbol{x}_{it} \Delta \boldsymbol{x}'_{it-1}\right) = K$ is fulfilled. However notice that the use of instruments $\boldsymbol{x}_{it-1}$ is only possible if $T \geq 3$
- For $T = 2$ we may use as instrument $\boldsymbol{x}_{it-1}$ but correlation between $\Delta \boldsymbol{x}_{it}$ and $\boldsymbol{x}_{it-1}$ is often small
- Efficient estimator in this context is the GMM, estimator which is exploiting all the sequential moment restrictions. However the properties of such estimator in small samples can be poor because of the large number of overidentifing restrictions .
- Notice as well that for uncorrelated u_{it} in base model we will observe the first order correlation for transformed error Δu_{it} in differenced model. This problem can be solved using clustered variance matrix.

Panel estimator in mixed case (some variables sequentially exogenous some strictly exogenous)

- It is possible that in the model

$$y_{it} = \boldsymbol{z}_{it}\gamma + \boldsymbol{w}_{it}\delta + c_i + u_{it}, \text{ for } t = 1, 2, \ldots, T$$

  where $\boldsymbol{z}_{it}$ is strictly exogenous but $\boldsymbol{w}_{it}$ is sequentially exogenous.
- In this case we estimate with $2SLS$ equation

$$\Delta y_{it} = \Delta \boldsymbol{z}_{it}\gamma + \boldsymbol{\Delta w}_{it}\delta + \Delta u_{it}, \quad \text{for } t = 2, \ldots, T$$

  using as instruments $\boldsymbol{z}_{it}, \boldsymbol{w}_{it-1}, \ldots, \boldsymbol{w}_{i,0}$ or any linear combination of them.

Typical application of this method is the estimation of model

$$y_{it} = \boldsymbol{z}_{it}\, \gamma + \rho_1 y_{it-1} + c_i + u_{it}$$

Estimation of the VAR model on panel data

- As all the explanatory variables variables are lags of the dependent variables they can be assumed to be sequentially exogenous if no autocorrelation is present in error term
- Then the *VAR* model can be estimated on panel data equation by equation using panel estimators for dynamic models
- Such estimation procedure is efficient given that we have no cross equation restrictions imposed on parameters (which is unusual)
- For such an estimated model we may calculate in a standard way *IRF*, *FEVD* and conduct Granger causality testing
- We can also calculate forecasts using standard methods although in this case we have to take into account individual effect when forecasting levels of the variables

Testing stationary using panel data

- Statistical model

$$y_{it} = \rho_i y_{it-1} + \boldsymbol{z}_{it}\gamma_i + u_{it}$$

  substracting from both sides y_{it-1} we obtain

$$\Delta y_{it} = \phi_i y_{it-1} + \boldsymbol{z}_{it}\gamma_i + u_{it}$$

- y_{it} is nonstationary if $\rho_i = 1$ (or equivalently for $\phi_i = 0$) for $i = 1, \ldots, N$
- $\boldsymbol{z}_{it}$ is consisting deterministic trends e.g. constant and trend $\boldsymbol{z}_{it} = [1, t]$
- There are several stationarity test, which differ with respect to null hypothesis on ρ_i (or ϕ_i), γ_i and assumptions made to obtain asymptotic distributions
- One aspect which is particularly important is wheter $H_0 : \rho_i = \rho = 1$ for all i or we assume that potentially $\rho_i \neq \rho_j$ and we test $H_0 : \rho_i = 1$ for all i

Testing stationarity using panel data cont.

- Asymptotic properties of estimators are mostly related to whether we consider asymptotics for $N \to \infty$ and T constant, for $T \to \infty$ and N constant, or for some combination of this assumptions
- Asymptotic distributions of the test are also dependent on deterministic elements included in model for y_{it}
- Tests (included in STATA)
  - Levin–Lin–Chu test
  - Harris–Tsavalis test
  - Breitung test
  - Im–Pesaran–Shin test
  - Fisher-type tests
  - Hadri LM test

Testing stationarity using panel data cont.

Properties of the test

Test	Trend	Asymptotics	ρ_i for H_1	Panel
LLC	no const	$\sqrt{N} \big/ T$	$\rho_i = \rho$	balanced
LLC		$N/T \to 0$	$\rho_i = \rho$	balanced
LLC	trend	$N/T \to 0$	$\rho_i = \rho$	balanced
HT	no const	$N \to \infty$, T const	$\rho_i = \rho$	balanced
HT		$N \to \infty$, T const	$\rho_i = \rho$	balanced
HT	trend	$N \to \infty$, T const	$\rho_i = \rho$	balanced
Breitung	no const	$(T, N) \to_{\text{seq}} \infty$	$\rho_i = \rho$	balanced
Breitung		$(T, N) \to_{\text{seq}} \infty$	$\rho_i = \rho$	balanced
Breitung	trend	$(T, N) \to_{\text{seq}} \infty$	$\rho_i = \rho$	balanced
IPS	trend	$N \to \infty$, T const	$\rho_i \neq \rho_j$	unbalanced
IPS	lags	$(T, N) \to_{\text{seq}} \infty$	$\rho_i \neq \rho_j$	unbalanced
IPS	trend with lags	$(T, N) \to_{\text{seq}} \infty$	$\rho_i \neq \rho_j$	unbalanced
Fisher		$T \to \infty$, N const	$\rho_i \neq \rho_j$	unbalanced
Hadri LM		$(T, N) \to_{\text{seq}} \infty$	$\times$	balanced
Hadri LM	trend	$(T, N) \to_{\text{seq}} \infty$	$\times$	balanced

Testing cointegration on panel data

- Data generating process

$$\Delta y_{it} = \alpha_i \left(y_{i,t-1} - \boldsymbol{\beta}_i \mathbf{x}_{i,t-1} \right) + \sum_{j=1}^{p_i} \boldsymbol{\alpha}_{ij} \Delta y_{i,t-j} + \sum_{j=-q_i}^{p_i} \alpha_{ij} \Delta x_{i,t-j} + \boldsymbol{\delta}_i \boldsymbol{d}_t + \varepsilon$$

- Transforming this equation we obtain

$$\Delta y_{it} = \alpha_i y_{i,t-1} - \boldsymbol{\lambda}_i \mathbf{x}_{i,t-1} + \sum_{j=1}^{p_i} \boldsymbol{\alpha}_{ij} \Delta y_{i,t-j} + \sum_{j=-q_i}^{p_i} \alpha_{ij} \Delta x_{i,t-j} + \boldsymbol{\delta}_i \boldsymbol{d}_t + \varepsilon_{ij}$$

  where $\boldsymbol{\lambda}_i = \alpha_i \boldsymbol{\beta}_i$
- If $\alpha_i = 0$, then there is no error correction and the cointegration is not present, if $\alpha_i < 0$ then for unit i the cointegration is present.
- The hypothesis of no cointegration can then be formulated as follows

$$H_0 : \alpha_i = 0 \text{ for } i = 1, \ldots, N$$

Testing cointegration on panel data cont.

- Alternative hypothesis can be formulated in two ways
  - if we assume that for some units α_i are different than for other units

$$H_1 : \alpha_i < 0 \text{ for some } i \qquad ((*))$$

  - if we assume that α_i are the same for all units

$$H_1 : \alpha_i = \alpha < 0 \text{ for all } i \qquad ((**))$$

- Statistics defined for case (*) are known as group statistics
- Statistics defined for case (**) are known as panel statistics

Testing cointegration on panel data cont.

- When calculating statistics we estimate with nonparametric methods long run covariance matrix
- In the case of both hypothesis we can either use the statistic based on statistics t (group G_τ, panel P_τ) or on statistics $T\widehat{a}$ (group G_α, panel P_σ).
- The choice of the statistic depends on the research problem
- Wersterlund derived the asymptotic distribution of this test for $(T, N) \rightarrow_{\text{seq}} \infty$