

House Prices, Real Estate Returns, and the Business Cycle

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Abstract

The main objective of this work is to construct a model that could be used to study investment dynamics, house prices, real estate returns and the equity premium within the same unified framework. The second objective is to show that the introduction of habit formation and residential real estate in the utility function provides an explanation of the puzzling volatility of house prices. To address these issues, we construct a two-sector model composed of a real estate sector and of a non-residential sector. The main contribution of this work is to show that an extended version of the standard neoclassical model is able to generate theoretical moments and risk premiums that are in line with the stylized facts.

- Keywords: house prices, real estate returns, equity premium
- JEL: E30, E22, G12

1 Introduction

According to data on gross domestic product by industry, in America, real estate accounts for 15% of total value added¹. Moreover, many more people own homes than own shares. In nearly all the big developed economies, well over half of all households are home-owners. In most of Europe and Australia, housing accounts for 40-60% of total household wealth, and in the United States for about 30%. And even in America the typical household on an average income holds six times as much wealth in residential property as in shares².

Yet, as far as explaining the behavior of asset prices is concerned, while the case of equity and bond prices has been extensively treated, when it comes to the relationship between real estate and the business cycle, very little is known. And whereas modelling the housing sector has become central to the process

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¹Source: Bureau of Economic Analysis (BEA)

²Source: The Economist, Survey Property: House of Cards, May 29th 2003

of macroeconomic forecasting, some key aspects of real estate market activity remain difficult to explain. As shown by Davis and Heathcote (2003), while multi-sector models can be quite successful at accounting for the high volatility of residential investment, explaining simultaneously this volatility and the fact that house prices are more volatile than output remains challenging³.

The work linking the macroeconomy to real estate returns is even more limited. While extended versions of the standard neoclassical model [see Jermann (1998), Boldrin, Christiano and Fisher (2001)] are able to generate an equity premium which is in line with the empirical evidence, there is little work studying real estate returns using a dynamic general equilibrium approach. Moreover, the dynamics of dividends and the volatility of investment being closely linked, introducing a residential sector in the standard model gives rise to an interesting question. In addition to the volatility of house prices, is it possible to build a model that could account for the fact that residential investment is twice as volatile as non-residential investment? Is it possible to reproduce these facts in a model that could also explain the risk premium on real estate returns and the equity premium? As noted by Leung (2004), very few studies have attempted to construct a model that could be used to study investment dynamics, house prices, real estate returns and the equity premium within the same unified framework.

The main objective of this paper is to contribute to close this gap. The second objective is to show that the introduction of habit formation and residential real estate in the utility function provides an explanation of the puzzling volatility of house prices. To address these issues, we construct a two-sector model [see Baxter (1996)] composed of a real estate sector and a non-residential or numeraire sector. Dividends in the non-residential sector can be used to study equity returns, and similarly dividends in the real estate sector are used to derive real estate returns. We then assess whether the model that is developed is able to account for the empirical facts. To be successful, the candidate model, in addition to the standard stylized facts, will thus have to be able to explain the risk premiums on both the residential and the non-residential sector, the low risk free rate, both residential and non-residential investment dynamics, the volatility of house prices, the volatility of real estate investment trust prices, the volatility of equity prices and the fact that the three assets under consideration are less correlated with output than the other macroeconomic variables.

The main contribution of this paper is to show that an extended version of the standard neoclassical model is able to generate theoretical moments and risk premiums that are in line with these stylized facts. As far as accounting for asset market fact is concerned, as in Jermann (1998), the key ingredients that allow the model to successfully reproduce the equity premium and the risk premium on real estate returns are the introduction of habit persistence in consumption and capital adjustment cost. When it comes to explaining the volatility of house prices, we find that adding a real estate sector producing residential real estate services, introducing land in the production function of

³In their models, house prices are found to be less than half as volatile than output.

the non-residential sector and allowing households to accumulate residential real estate over time are key modifications to the standard neoclassical model.

2 Relevant Literature

Following the literature on asset pricing in production economies initiated by Jermann (1998) [see also Jermann (1994)], Boldrin, Christiano and Fisher (2001) show that a model with limited sectorial mobility is also able to explain the equity premium and the risk free rate while allowing labor to be determined endogenously. When it comes to the link between the equity premium puzzle and house prices, Piazzesi, Schneider and Tuzel (2003) show that adding housing services in the utility function helps to generate asset market implications that are more consistent with the empirical facts.

As far as the distinction between residential and commercial real estate is concerned, the model developed by Kan, Kwong and Ka-Yui Leung (2000) allows to differentiate between the two assets. However, they only consider the interaction between prices and output and the implications of the model in terms of asset pricing are not developed. Moreover, in contrast to the traditional literature on business cycles, the behavior of other important variables related to the cycle such as investment are not studied.

Some recent contributions have investigated the link between house prices and the macroeconomy in artificial economies subject to credit constraint. Since Bernanke and Gertler (1989), various authors, including Kiyotaki and Moore (1997), Calstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (2000) have presented dynamic micro founded models where frictions in the financial markets exacerbate output fluctuations in response to aggregate disturbances. Following this literature, Iacoviello (2003) introduces real estate in an economy composed of agents facing binding borrowing constraints and where real estate plays the role of a collateral used to secure loans. Similarly, Aoki, Proudman and Vliegh (2002) construct a financial accelerator model where house prices amplify fluctuations in consumption and housing investment over the business cycle. As explained by Iacoviello (2003), this literature is based on the view that deteriorating credit market conditions, such as growing debt burden and falling asset prices, are not simply passive reflections of a declining economy, but are themselves a major factor depressing the economy.

However, as noted by Iacoviello (2003), while it is accepted that capital market imperfections matter for economic activity, there is still controversy about the quantitative relevance of these frictions at the aggregate level. In a recent study discussing the main forces that are the most likely to have caused the Great Depression, Chari, Kehoe and McGrattan (2003) reach the conclusion that, to the extent that monetary shocks drove the Great Depression, the mechanisms advocated by Bernanke and Gertler (1989) could explain neither the important fall in output from 1929 to 1933 nor the recovery after 1933 that have been documented. While the debate is still ongoing, according to the results of Chari, Kehoe and McGrattan (2003), it seems that building models

based on credit market imperfections is not likely to yield a very high payoff.

Following the home production literature [see Benhabib, Rogerson and Wright (1991), Greenwood and Hercowitz (1991)] in addition to the multi-sector growth literature that begins with Long and Plosser (1983), Davis and Heathcote (2003) build a neoclassical multi-sector stochastic growth model in order to explain the dynamics of residential investment. While successful in explaining the behavior of the main macroeconomic aggregate as well as the behavior of residential and non-residential investment, the model developed by these authors fails to account for the observed volatility of house prices in the USA. In their model, house prices are found to be less than half as volatile as output while in the data house prices are more volatile than output.

As for residential investment and house prices, Jin and Zeng (2003) develop a three-sector model driven by three different productivity shocks and one monetary shock. While the model is quite successful at accounting for some of the salient business cycle properties concerning residential investment and house prices, the fact that a large number of exogenous shocks are needed to generate these conclusions is somewhat unsatisfying. Moreover, the dynamics of some key macroeconomic variables such as aggregate consumption are not studied.

There is another strand of literature that considers the role of housing in incomplete market environments. These models typically either focus on steady states [see Platania and Schlagenhauf (2000) or Fernandez-Villaverde and Krueger (2002)] or else abstract from the production side of the economy [see Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992), Peterson (2003), and Ortalo-Magne and Rady (2001)]. But, as explained by Davis and Heathcote (2003), while frictions such as poorly functioning rental and mortgage markets are likely to be important in accounting for cross-sectional issues, it is not obvious that they are important for housing dynamics at the aggregate level.

As for empirical evidence on the determinants of real estate returns, McCue and Kling (1994), employing an unrestricted vector autoregressive model reach the conclusion that macroeconomic variables explain approximately 60% of the variations in real estate returns. However, while shocks to nominal interest rates are found to have a significant negative influence on returns, the impact of other sources of fluctuations that are often used in the macroeconomic literature, such as productivity shocks, are not investigated [see Christiano, Eichenbaum and Vigfusson (2003)].

3 Stylized Facts, Questions and Assumptions

As proposed by Prescott (1986), one way of evaluating the predictions of a theoretical framework is to compare moments that summarize the actual experience of an economy with similar moments from the model. In our context, the key moments to replicate are presented in table 1, where \hat{y}_t^T , \hat{c}_t , \hat{i}_t^r , \hat{i}_t^{nr} and \hat{p}_t^h denote respectively output, consumption, residential investment, non-residential investment and house prices. House price data are provided by the Bank for

International Settlement⁴. The remaining series can be found on the website of the Federal Reserve Bank of St-Louis. All series are available quarterly. To generate the empirical moments, the variables are, firstly, deflated using CPI inflation and expressed in logarithm. Secondly, to be able to compare the data with the model solution (see the transformed model in the technical appendix, section 11.1), the deterministic trend of output is removed from all series⁵. The cyclical component is then computed by subtracting the logarithm of the HP filtered series from the logarithm of the actual detrended series. We denote variables that have been subject to this transformation by a hat.

Second, following McCue and Kling (1994), we use Real Estate Investment Trust (REIT) data as a proxy for real estate returns. The NAREIT Real-Time index, which includes all real estate investment trust currently trading on the New York Stock Exchange, the NASDAQ and the American Stock Exchange and for which monthly historical statistics from 1972 are available, is used. The advantage of working with financial data is that price indices as well as total return indices are provided. Price indices and total return indices are available from DATASTREAM. We denote by \hat{p}_t^r the cyclical component of the price index describing the evolution of real estate investment trust prices and by r^r the average total return. Similarly, we use the MSCI equity index as a proxy for equity prices of the non-residential sector and \hat{p}_t^e denotes the cyclical component of the price index whereas r^e is the average total return. The average risk free rate, r_f , is computed using the three month treasury bill rate and CPI inflation⁶. The risk premiums on equity returns and on real estate returns are respectively denoted by π^e and π^r .

Table 1: Empirical Moments USA (1970-2002)⁷

<i>Empirical Moments</i>	$\sigma_{\hat{x}_t}$	$\frac{\sigma_{\hat{x}_t}}{\sigma_{\hat{y}_t^T}}$	$corr(\hat{x}_t, \hat{y}_t^T)$
\hat{y}_t^T	1.58	1	1
\hat{c}_t	1.28	0.81	0.85
\hat{i}_t^r	10.52	6.66	0.73
\hat{i}_t^{nr}	5.03	3.18	0.80
\hat{p}_t^h	2.16	1.37	0.65
\hat{p}_t^e	11.53	7.3	0.44
\hat{p}_t^r	11.9	7.53	0.46

⁴ Collected from national source

⁵ The linear trend extraction has no impact on the variance and the correlation with output of the HP filtered series.

⁶ Source: Federal Reserve Bank of St-Louis for the three month treasury bill rate and CPI inflation

⁷ $\sigma_{\hat{x}_t}$ denotes the standard deviation of the cyclical component of the serie, and $\sigma_{\hat{x}_t}/\sigma_{\hat{y}_t^T}$ the relative standard deviation of the serie with respect to output.

**Table 2: Risk Free Rate, Real Returns and Risk Premiums USA
(1970-2002)**

	Risk Free Rate (Annualized%)		
r^f	1.45		
	Real Returns (Annualized%)		Risk Premiums (Annualized%)
r^e	7.41	π^e	5.96
r^r	7.29	π^r	5.84

As can be seen in table 1, while residential investment, \hat{i}_t^r , is more than 6 times as volatile as output and twice as volatile as non-residential investment, \hat{i}_t^{nr} ; house prices, \hat{p}_t^h , are only slightly more volatile than output. Moreover, while the correlation between house prices \hat{p}_t^h and output is positive, house prices are less correlated with output than the main macroeconomic variables. Real estate investment trust prices, \hat{p}_t^r and equity prices, \hat{p}_t^e , are also significantly less correlated with output than consumption and both residential and non-residential investment.

In the context of production economies with a representative non-residential sector [see Jermann (1998)], the equity premium puzzle stems from the fact that non-residential investment is highly volatile. In a competitive equilibrium, dividends being defined as the capital share of output less investment⁸, following a productivity shock, while consumption respond positively, the significant increase in non-residential investment offset the positive effect that the shock has on dividends and as a result dividends respond negatively. This negative co-movement between consumption and dividends (or positive co-movement between marginal utility and dividends) that is generated by the standard model implying that the risk associated with holding equities is low, is hard to reconcile with the empirical facts and with an equity premium as high as 5.96% (see table 2).

When it comes to explaining the risk premium on real estate returns, π^r , and the high volatility of residential investment, \hat{i}_t^r (see table 1 and 2), integrating a real estate sector into the analysis gives rise to some serious complications. Firstly, the difficulty to account for the high risk premium will be even greater since the negative impact on real estate dividends caused by the large response of residential investment will be even more pronounced. The negative co-movement between consumption and real estate dividends should thus be even greater than in the non-residential case. However, as reported in table 2, despite the significant differences in residential and non-residential investment dynamics, the risk premium on real estate returns and the equity premium are almost identical.

⁸In the competitive equilibrium, dividends are given by: $d_t = y_t - W_t N_t - i_t = \xi y_t - i_t$ where ξ denotes the capital share of output in the production function.

To be successful, the candidate model, in addition to the standard stylized facts, will thus have to be able to explain the risk premiums on both the residential and the non-residential sector, the low risk free rate, both residential and non-residential investment dynamics, the volatility of house prices, the volatility of real estate investment trust prices, the volatility of equity prices and the fact that the three assets under consideration are less correlated with output than the other macroeconomic variables.

While introducing sector specific productivity shocks, is often convenient to account for the differences in dynamics describing two sectors [see Baxter (1996)], in this work it will be assumed that the technology available is the same for each representative firms in the two sectors and that technology shocks are the dominant source of business cycle fluctuations. The real estate industry being composed of many different activities ranging from residential services to construction, we find more natural to assume that the non-residential good producer and the real estate service producer are equally affected by technological change and that within the same country there are no barriers to technology adoption. We thus restrict the analysis to the existence of one single aggregate technology shock. The differences in dynamics that the model will produce for the two sectors will therefore not arise from the specification of the exogenous process.

4 The Model

The economy is composed of a representative agent, as well as two sectors, a numeraire or non-residential sector and a real estate or residential sector, denoted respectively by e and r . The first sector produces a consumption good while the second sector produces a composite real estate good. The numeraire sector uses capital, labor and land to produce, and capital is owned by the firm. As for the real estate sector, following Davis and Heathcote (2003), the final real estate good is produced using residential structures and labor as inputs. In both sectors, the accumulation of the capital stock and of the residential structure stock can be financed through retained earnings. There is an initial endowment of land owned by the real estate service producer and which can be rented to the firms in the non-residential sector.

To ensure steady state growth, technical progress, Γ_t , is introduced and is assumed to take a labor augmenting form [see King, Plosser and Rebelo (2002)] and the transformed economy can be defined in terms of stationary variables (see the technical appendix, section 11.1). The law of motion for Γ_t which denotes the deterministic component of growth evolves according to:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma$$

4.1 The Consumers

As far as defining preferences is concerned, as in the standard case, the representative consumer firstly receives utility from consuming the numeraire consumption good and secondly from the consumption of services from the real estate good. Following the literature on habit formation we assume that agents can accumulate stocks of both the consumption good and of the real estate composite good. Formally, both stocks evolves according to the following laws of motion:

$$\gamma x_{t+1}^c = a^c x_t^c + b^c c_t \quad (1)$$

$$\gamma x_{t+1}^h = a^h x_t^h + b^h h_t^r \quad (2)$$

where c_t and h_t^r denote respectively the service flow of a standard consumption good and the service flow of a composite real estate good and γ is the quarterly growth trend rate (see the technical appendix, section 11.1). We define x_t^h as the stock of residential real estate. The stock of residential real estate in period $t+1$ is given by the sum of the stock of the current period, x_t^h , adjusted for depreciation, a^h , and the service flow of the real estate good of the current period, h_t^r . The parameter b^h , where $0 \leq b^h \leq 1$, represents the share of the composite real estate good that can be used to increase the stock of residential real estate held by the household.

Similarly, x_{t+1}^c , denotes the stock of the consumption good accumulated by the agent. This specification of what we interpret as accumulation of the consumption good is equivalent to the specification adopted in Boldrin, Christiano and Fisher (2001) or Fuhrer (2000) where the evolution of the stock of habit is defined as in (1).

The representative agent maximizes expected lifetime utility given by:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} u_t(c_t, x_t^h, h_t^r, x_t^c) \right\}$$

where:

$$u_t(c_t, x_t^h, h_t^r, x_t^c) = \frac{1}{1-\sigma} [(c_t - m^c x_t^c)^\kappa (h_t^r - m^h x_t^h)^{1-\kappa}]^{1-\sigma}$$

and where β^* is the modified subjective rate of time discount (see the technical appendix, section 11.1) and σ the coefficient of risk aversion.

This specification of utility will allow us to consider several interesting cases. First, if m^c and m^h are set to zero, as in the standard case with leisure, both goods are substitutes [see Baxter and Crucini (1993), Jermann (1994)]. The sign of the two coefficients m^c and m^h will allow us to choose between habit and durability. If the two coefficients are positive, the agent develops habits for higher or lower consumption of c_t and h_t^r . In contrast, in the case of durability, the coefficients are negative, and instead of depending on the difference between

current and past levels, current utility will continue to be positively influenced by levels of consumption taken during past periods.

When it comes to choosing the value of these parameters, some interpretation can be helpful. In terms of this notation, the case studied in Jermann (1998) where utility is defined as:

$$\frac{1}{1-\sigma} [(c_t - (m^c/\gamma)b^c c_{t-1})]^{1-\sigma}$$

corresponds to the case $m^c = 1$ and $a^c = 0$. Consumption being a perishable good, such an assumption which implies that the stock of consumption completely depreciates within the period seems the most appropriate and will be adopted in this paper. The case $m^c = 1$ and $a^c, b^c \neq 0$ corresponds to the specification adopted in Boldrin, Christiano and Fisher (2001). When it comes to the stock of the real estate good, x_t^h being interpreted as the stock of residential real estate, given that, in contrast to perishable consumption, homes depreciate slowly, it seems that choosing a value close to 1 for a^h will be more appropriate. As for b^h , this parameter can be interpreted as the sensitivity of the stock of residential real estate with respect to the consumption of real estate services. The fraction of the real estate good given by $(1 - b^h)h_t^r$ can thus be seen as consumption of real estate services that have no impact on the stock of housing such as expenditures related to maintenance. In contrast, the fraction spent on the real estate good given by $b^h h_t^r$ allows the household to increase its stock of housing. Given that constructing new homes is time consuming, we can think that compared to the stock of a standard consumption good, adjusting the stock of residential real estate will be particularly costly and so that b^h will be low in comparison with b^c . Low values of b^h will therefore imply that increasing the stock of residential real estate require the agent to buy large amounts of the real estate good and thus will capture the idea that increasing the stock of residential real estate is costly.

4.2 The Problem of the Consumer

In the stationary economy, where detrended variables are denoted by lower case letters⁹, the representative household chooses each period its level of consumption, c_t , how much of the real estate good, h_t^r , to buy from the real estate service producer at the price z_t^r , the number of hours to work in both sector, N_t^e, N_t^h , and how many equities of the real estate service producer, S_{t+1}^r , and of the consumption good producer, S_{t+1}^e , to hold. The equity price of the real estate service producer and of the consumption good producer is denoted respectively by p_t^r and p_t^e . A labor income $W_t^e N_t^e$ and $W_t^h N_t^h$ as well as a capital income, represented by the dividends, d_t^e, d_t^h , are paid each period by both firms to the representative household. We assume for simplicity that leisure does not enter in the utility function and that the household allocate a fixed fraction of its time to either the real estate sector or the numeraire sector.

⁹For example, detrended consumption is expressed as: $c_t = C_t / \Gamma_t$

The problem of the representative household is defined by the following program:

$$\begin{aligned} \text{Max } & E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} u_t(c_t, x_t^h, h_t^r, x_t^c) \right\} \\ \text{s.t. } & W_t^h N_t^h + W_t^e N_t^e + S_t^e (p_t^e + d_t^e) + S_t^r (p_t^r + d_t^r) = c_t + z_t^r h_t^r + p_t^e S_{t+1}^e + p_t^r S_{t+1}^r \quad (3) \end{aligned}$$

$$a^c x_t^c + b^c c_t = \gamma x_{t+1}^c \quad (4)$$

$$a^h x_t^h + b^h h_t^r = \gamma x_{t+1}^h \quad (5)$$

$$t^r = l_t^r + n_t^r \quad (6)$$

$$t^e = l_t^e + n_t^e \quad (7)$$

where t^r and t^e denote initial endowments of time that the agent allocate to each sector.

4.3 Asset Prices

The first-order conditions with respect to S_{t+1}^e and S_{t+1}^r define respectively the dynamics of equity prices of the consumption good sector and equity prices of the real estate sector.

$$\begin{aligned} S_{t+1}^e : \\ \lambda_t p_t^e = \beta^* E_t \lambda_{t+1} [d_{t+1}^e + p_{t+1}^e] \end{aligned} \quad (8)$$

$$\begin{aligned} S_{t+1}^r : \\ \lambda_t p_t^r = \beta^* E_t \lambda_{t+1} [d_{t+1}^r + p_{t+1}^r] \end{aligned} \quad (9)$$

where λ_t is the lagrange multiplier associated with constraint (3). These two conditions are standard and can be interpreted using the central asset pricing formula applied to equity prices. Given the payoff $d_{t+1} + p_{t+1}$, and given the investor's consumption choice λ_t , this condition tells us what market price p_t to expect. $\lambda_t p_t$ is the loss in utility if the agent buys another unit of the asset; $\beta^* E_t \lambda_{t+1} [d_{t+1} + p_{t+1}]$ is the increase in expected utility he or she obtains from the extra payoff at $t+1$. The agent continues to buy or sell the asset until the marginal loss equals the marginal gain ¹⁰.

The conditions describing the dynamics of the stock of residential real estate and of the stock of numeraire consumption are given by:

$$\begin{aligned} x_{t+1}^h : \\ \phi_t^h = \tilde{\beta}^* a^h E_t \phi_{t+1}^h + \tilde{\beta}^* \frac{\partial u_{t+1}(c_{t+1}, x_{t+1}^h, h_{t+1}^r, x_{t+1}^c)}{\partial x_{t+1}^h} \end{aligned} \quad (10)$$

¹⁰ See Cochrane (2001) for a more detailed interpretation of this condition

x_{t+1}^c :

$$\phi_t^c = \tilde{\beta}^* a^c E_t \phi_{t+1}^c + \tilde{\beta}^* \frac{\partial u_{t+1}(c_{t+1}, x_{t+1}^h, h_{t+1}^r, x_{t+1}^c)}{\partial x_{t+1}^c} \quad (11)$$

where ϕ_t^h and ϕ_t^c are the lagrange multipliers associated with constraints (4) and (5). ϕ_t^h and ϕ_t^c can alternatively be interpreted as the shadow price of respectively residential real estate and of the stock of numeraire consumption. As shown in the technical appendix (section 11.5), exploiting the equivalence between the competitive equilibrium and the problem of the social planner allows us to derive two equivalent definitions of house prices. House prices can, firstly, be defined as the relative shadow price of residential real estate with respect to the stock of numeraire consumption:

$$p_t^h = \frac{\phi_t^h}{\phi_t^c} \quad (12)$$

or equivalently as the infinite discounted sum of future expected marginal utility of residential real estate [see equation (64) in the technical appendix]¹¹.

As for equation (10) and (11), compared to the previous conditions describing the evolution of equity prices, the subjective time discount factor attached to the above conditions $\tilde{\beta}^*$ is adjusted such as to account for the fact that in contrast to the stock of equity which is fixed, the stock of residential real estate and of numeraire consumption vary over the business cycle. The implication of this definition in terms of the dynamics of house prices and its sensitivity to the parameter values describing the law of motion of the stock of habit and of the stock of residential real estate will be discussed later.

4.4 The Problem of the Firms

This section describes the problem of the two representative firms in both the numeraire and the real estate sector. Compared to the standard model [see King, Plosser and Rebelo (1989)], we introduce capital adjustment cost in both sectors and land is owned by the firms in the real estate sector. The two final goods produced by the economy are a numeraire consumption good in the non-residential sector and a composite real estate good in the residential sector.

4.4.1 The Numeraire Sector

In each period, the numeraire good producer has to decide how much labor to hire, N_t^e , how many units of land, h_t^c , to rent from the real estate service producer and how much to invest, i_t^e . Managers maximize the value of the firm

¹¹ where the marginal utility of residential real estate is given by:

$$\frac{\partial u_t(c_t, x_t^h, h_t^r, x_t^c)}{\partial x_t^h}$$

to its owners, the representative agent, and is equal the present discounted value of all current and expected cash flows d_t^e :

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^{*t} \lambda_t d_t^e$$

with:

$$d_t^e = y_t^e - W_t^e N_t^e - z_t^c h_t^c - i_t^e \quad (13)$$

and where $\beta^{*t} \lambda_t$ is the marginal rate of substitution of the owner. y_t^e denotes output, W_t^e the wage rate paid to the workers and z_t^c the rental cost of land. The firm's capital stock follows an intertemporal accumulation equation with adjustment cost and varying rate of capital utilization:

$$(1 - \delta^e) k_t^e + \Phi^e(\frac{i_t^e}{k_t^e}) k_t^e = \gamma k_{t+1}^e \quad (14)$$

As in Baxter and Crucini (1993), the parameters of the capital adjustment costs function $\Phi^e(\frac{i_t^e}{k_t^e})$ are set so that the model with adjustments costs has the same steady state as the model without adjustments costs and it is assumed that near the steady state point: $\Phi^e > 0$, $\Phi^{e'} > 0$ and $\Phi^{e''} < 0$. This captures the idea that increasing the capital stock rapidly is more costly than changing it slowly. The numeraire sector require the use of labor, N_t^e , capital, k_t^e , and land, h_t^c . The good is produced via Cobb-Douglas production function:

$$y_t^e = A_t k_t^{e\xi} N_t^{e\alpha} h_t^{c(1-\alpha-\xi)} \quad (15)$$

where A_t is the standard random productivity shock variable that can be interpreted as a temporary displacement to total factor productivity, and which is identical across sector.

The first-order conditions are presented in the technical appendix (see section 11.4).

4.4.2 The Real Estate Sector

The problem of the real estate good producer is similar to the problem of the numeraire good producer. The use of labor, N_t^r , and residential structures, k_t^r , is required to produce the composite real estate good and the firm is endowed with a fixed amount of land that is rented to the numeraire good producer at the rental price z_t^c . The dividend of the firm producing the real estate good, expressed in nominal terms, is given by:

$$d_t^r = z_t^r y_t^r + z_t^c h_t^c - z_t^r i_t^r - W_t^r N_t^r \quad (16)$$

The production function is given by:

$$y_t^r = A_t k_t^{r\rho} N_t^{r(1-\rho)} \quad (17)$$

The problem of the real estate service producer can be described by the following program:

$$\begin{aligned} \text{Max } & E_0 \sum_{t=0}^{\infty} \beta^{*t} \lambda_t d_t^r \\ \text{s.t. } & d_t^r = z_t^r y_t^r + z_t^c h^c - z_t^r i_t^r - W_t^r N_t^r \\ & (1 - \delta^r) k_t^r + \Phi^r \left(\frac{i_t^r}{k_t^r} \right) k_t^r = \gamma k_{t+1}^r \end{aligned}$$

where the subscript r denotes variables used to describe the real estate sector. The first-order conditions are presented in the technical appendix (see section 11.3).

5 Market Clearing Conditions

The quantity of equity supplied by each sector being constant and normalizing it to one, in each sector, the market clearing condition on the equity market are:

$$S_t^r = S_{t+1}^r = 1 \quad (18)$$

and:

$$S_t^e = S_{t+1}^e = 1 \quad (19)$$

for the real estate sector and for the numeraire sector.

As far as the labor market is concerned, since utility is strictly increasing in the consumption good and in the real estate good and leisure does not yield utility, in equilibrium, we will have:

$$N_t^e = \overline{N^e} \quad (20)$$

and:

$$N_t^r = \overline{N^r} \quad (21)$$

So labour will be supplied inelastically by the representative household.

Finally market clearing in the numeraire sector implies that:

$$y_t^e = c_t + i_t^e \quad (22)$$

and similarly, in the real estate sector, we have that:

$$y_t^r = h_t^r + i_t^r \quad (23)$$

6 Risk Premiums

As far as the risk premiums are concerned, as shown by Jermann (1994), the conditional expected return of a claim to a dividend that will be paid in k periods can be expressed as:

$$E_t(R_{t,t+1}[D_{t+k}]) = R_{t,t+1}[1_{t+1}] \times \exp[-\text{cov}_t(\lambda_{t+1}, E_{t+1}\lambda_{t+k} - \lambda_{t+1})] \\ \times \exp[-\text{cov}_t(\lambda_{t+1}, E_{t+1}d_{t+k})] \quad (24)$$

where D_{t+k} denotes the payoff associated with an asset offering a gross return of $R_{t,t+1}[D_{t+k}]$, λ_{t+k} and d_{t+k} the marginal utility of consumption and dividends in logs. The first term represents the return to holding a one-period bond until maturity, *i.e.*, the risk free rate. The second term can be thought of as a holding or term premium for a k -periods discount bond that depends on the term structure of interest rates. If one expects capital gains as a consequence of lower interest rates when consumption is high, then this unfavorable correlation has to be compensated by a positive risk premium. The last element, the payout uncertainty premium, is linked to a possible capital gain or loss at time $t+1$. If capital gains are negatively correlated with the valuation, then a risk premium is needed to compensate the investor for the undesirable cyclical property of this asset. The above formula can be rewritten as:

$$E_t(R_{t+1}[D_{t+k}]) = R_{t,t+1}[1_{t+1}] \exp(\eta_h(k) + \eta_p(k)) \quad (25)$$

where $R_{t,t+1}[1_{t+1}]$ is the risk free rate, $\eta_h(k)$ the term premium and $\eta_p(k)$ the payout uncertainty premium.

Using this formulation, and considering an asset which represents a claim to an infinite sequence of dividends:

$$V_t[\{D_{t+k}\}_{k=1}^{\infty}] = \sum_{k=1}^{\infty} V_t[D_{t+k}]$$

the expected return of a common stock can be derived and is given by:

$$E_t(R_{t,t+1}[\{D_{t+k}\}_{k=1}^{\infty}]) = R_{t,t+1}[1_{t+1}] \sum_{k=1}^{\infty} w_t[D_{t+k}] \exp(\eta_h(k) + \eta_p(k)) \quad (26)$$

with $w_t[D_{t+k}]$ being the portfolio weight attached to the date $t+k$ dividend strip return. The equity premium is therefore given by:

$$RP[\{D_{t+k}\}_{k=1}^{\infty}] = \ln \left\{ \sum_{k=1}^{\infty} w_t[D_{t+k}] \exp(\eta_h(k) + \eta_p(k)) \right\} \quad (27)$$

using the definition of dividends in the real estate sector given earlier, this framework can be used to derive a similar relation for real estate returns (see the technical appendix, section 11.10).

7 Calibration

The objective of the quantitative evaluation is to derive the theoretical moments implied by the model and to compare them to the set of stylized facts presented earlier. The calibration is carried out in two steps. A first set of parameters is chosen based on National Income Account data, following the standard in the business cycle literature. A second set of parameters, for which precise a priori knowledge is weak, is chosen to maximize the model's ability to replicate the set of business cycle and asset prices moments.

- Long -run behavior

A first set of parameters is chosen to match long-run model behavior. These parameters do not significantly affect the model dynamics and standard values are used. The share of consumption in the utility function κ is set to 1/2. As in Jermann (1998), the quarterly trend growth rate γ is 1.005, and the capital depreciation rate δ^e is 0.025. In order to maximize the model's ability to replicate the low risk free rate, the modified discount factor β is set to 0.9975. The constant labor share in the Cobb-Douglas production function of the numeraire good producer α is 0.66, the capital share ξ is 0.23 which implies a share of land of 0.11. As for the production function of the real estate service producer, the residential structures share is fixed to 0.13. Following Davis and Heathcote (2003), the rate of depreciation of residential structure, δ^r , is 0.004.

- Aggregate output

Aggregate output is defined as the weighted average of output in both sectors. Using data on the component of value added by industry, we set the share of the real estate sector to 0.15.

- Technology shocks

The parameters describing the technology shocks are chosen such as to replicate U.S. postwar quarterly output growth volatility. The standard deviation of the shock innovation σ_ε^2 is 1.15 percent. The persistence parameter ρ_A is 1.

- Coefficient of risk aversion

Estimates for this parameter go from about 2 in Friend and Blume (1975) to values as high as 21 in Campbell (1993). In Mehra and Prescott (1985), it is argued that values for this parameter ranging from 1 to 10 could be considered as reasonable. Drawing on Kocherlakota (1996)'s review, we use $\sigma = 3$.

- Habit Formation vs Durability

Following many studies [see Jermann (1998); Boldrin, Christiano and Fisher (2001); Fuhrer (2000)], we assume habit formation in consumption and set m^c to -1. As for real estate, given the specification adopted, where in contrast to

the consumption good, the stock of residential real estate does not depreciate completely within the period, we assume that utility is derived from both the consumption of the composite real estate good h_t^r and the current stock of real estate x_t^h . We thus set m^h to -1 , which is equivalent to assume durability in real estate.

- Habit persistence and accumulation of residential real estate

Given that the empirical literature do not seem to offer much precise guidance when it comes to calibrating habit formation and the accumulation of residential housing stock, we make use of the economic interpretation of these parameters to choose their values (see sections 4.1 and 8.2). Following Jermann (1998), we set a^c to 0 and b^c to 0.82. As far as the equation describing the accumulation of residential housing is concerned, in our benchmark calibration, a^h is 0.995, the depreciation rate of residential real estate is thus 0.005. The parameter b^h is set to 0.004.

- Capital adjustment costs

As in Baxter and Crucini (1993), the parameters of the capital adjustment costs function $\phi(\frac{i_t}{k_t})$ are set so that the model with adjustments costs has the same steady state as the model without adjustments costs and it is assumed that near the steady state point: $\phi > 0$, $\phi' > 0$ and $\phi'' < 0$. This captures the idea that increasing the capital stock is costly. As a result, to reach a given level of capital stock, compared to the situation without adjustment cost, more investment will be necessary as the adjustment cost function imply that part of the increase in investment is wasted. The parameter c represents a measure of the curvature of the adjustment cost function (see the technical appendix, section 11.6). Thus, the higher c the higher is the marginal cost of increasing the investment to capital ratio. For the non-residential sector, we set c^e to 1/0.36 and c^r to 0.13 for the real estate sector. These parameter values are discussed below.

7.1 Comparing Parameter Values

When possible, the calibration that is used to generate the results follows the one adopted in existing studies. The parameter values that are chosen in the non-residential sector and the driving process are similar to the one used in Jermann (1998).

- The non-residential sector

	Real Rigidities ¹²		
	a^c	b^c	c^e
Jermann (1998)	0	0.82	1/0.23
Two-sector model	0	0.82	1/0.36

¹² where a^c, b^c and c^e denote respectively the persistence of habit formation, the persistence of consumption and the elasticity of the capital adjustment cost function.

Long-run behavior and risk aversion¹³

	$\beta^{1-\sigma}$	δ^e	γ	ξ	α	σ
Jermann (1998)	0.99	0.025	1.005	1/3	2/3	5
Two-sector model	0.9876	0.025	1.005	0.23	2/3	3

- The driving process

Productivity Shocks¹⁴

	σ_ε^2	ρ_A
Jermann (1998)	0.01	0.99
Two-sector model	0.0115	1

As far as the capital share ξ is concerned, the difference is due to the fact that in our setting there is an additional production factor, land. As for the driving force, the values that are used for the shock innovation and the persistence parameter are slightly larger, which will allow us to match the volatility of output more accurately. As for $\beta^{1-\sigma}$ and c^e the differences in calibration arise from the fact that in Jermann (1998), following Mehra and Prescott (1985), the risk free rate and the average equity premium to be reproduced by the model are 0.81% and 6.18% whereas in our sample 1.45% is found for the risk free rate and 5.96% for the equity premium. To account for these facts, a slightly lower value for the adjustment cost parameter and for the modified discount factor are needed.

As for the real estate sector, when available, we choose parameter values very similar to what is done in Davis and Heathcote (2003). δ^r is 0.04, the labor share parameter $1 - \rho$, given that the real estate sector is more labor intensive than the residential sector is chosen so that $1 - \rho > \alpha$ in order to capture this fact, and is set to 0.87. The share of land $1 - \alpha - \xi$ is 0.11. In Davis and Heathcote (2003) where real estate developers combine land and residential structures to produce new houses, the share of land is 10.6. As far as the equation describing the accumulation of residential housing is concerned, in our benchmark calibration, a^h is 0.995, the depreciation rate for houses is thus 0.005, and b^h is 0.004. This calibration therefore implies that residential real estate depreciates slightly faster than residential structure ($\delta^h = 0.004$), which can be justified by saying that once houses are occupied, the value of existing homes is likely to depreciate faster. Finally, a low value for c^r is chosen in order to account for the high volatility of residential investment. The small value used for b^h is aimed at capturing the fact that, in contrast to the stock of numeraire consumption, increasing the stock of residential real estate is costly.

¹³where $\beta^{1-\sigma}, \delta^e, \gamma, \xi, \alpha$ and σ denote the modified discount factor, the rate of capital depreciation, the quarterly trend growth rate, the capital and the labor share in the production function and risk aversion.

¹⁴where σ_ε^2 is standard deviation of the shock innovation and ρ_A the persistence parameter.

8 Simulations and Results

Following the real business cycle literature, one way of evaluating the prediction of a model is to compare moments that summarize the actual experience of an economy with similar moments from the model. Table 3 reports summary statistics on HP-filtered cyclical components of key variables for simulations of the model driven by technology shocks. The following table presents the simulation results by showing the moments generated by the model that can be compared with the empirical moments shown earlier and that are repeated to facilitate comparisons.

Table 3: Theoretical Moments¹⁵ vs Empirical Moments

Empirical Moments	$\sigma_{\hat{x}_t}$	$\frac{\sigma_{\hat{x}_t}}{\sigma_{\hat{y}_t^T}}$	$corr(\hat{x}_t, \hat{y}_t^T)$	Theoretical Moments	$\sigma_{\hat{x}_t}$	$\frac{\sigma_{\hat{x}_t}}{\sigma_{\hat{y}_t^T}}$	$corr(\hat{x}_t, \hat{y}_t^T)$
\hat{y}_t^T	1.58	1	1	\hat{y}_t^T	1.58	1	1
\hat{c}_t	1.28	0.81	0.85	\hat{c}_t	1.41	0.89	0.92
\hat{i}_t^r	10.52	6.66	0.73	\hat{i}_t^r	10.61	6.73	0.75
\hat{i}_t^{nr}	5.03	3.18	0.80	\hat{i}_t^{nr}	4.21	2.67	0.73
\hat{p}_t^h	2.16	1.37	0.65	\hat{p}_t^h	2.16	1.37	0.66
\hat{p}_t^e	11.53	7.3	0.44	\hat{p}_t^e	11.92	7.56	0.70
\hat{p}_t^r	11.9	7.53	0.46	\hat{p}_t^r	11.00	6.98	0.64

Table 4: Theoretical vs Empirical Risk Premiums¹⁶

	Empirical Risk Premium (Annualized%)	Theoretical Risk Premium (Annualized%)
π^e	5.96	5.96
π^r	5.84	5.65

Table 5: Theoretical vs Empirical Risk Free Rate¹⁷

	Empirical Risk Free Rate (Annualized%)	Theoretical Risk Free Rate (Annualized%)
r^f	1.45	1.33

¹⁵ Log linear business cycle moments are the average over 5000 simulations

¹⁶ The risk premiums are the average over 200 simulations, each 3000 periods long.

¹⁷ The risk free rate is computed as the average of ten sample where in each sample 100000 observations have been simulated.

8.1 Residential and Non-residential Investment

As we can see in table 3, the model is able to account for the high volatility of residential investment and the fact that it is more than twice as volatile as non-residential investment. As in Davis and Heathcote (2003), introducing different depreciation rates for non-residential capital and residential structures largely contributes to account for these differences in volatility. By implying a higher real rate of return, slower depreciation in the real estate sector gives more incentives to invest in booms when productivity is high whereas in recession the fact that existing structures depreciate more slowly, implies that firms need to save a smaller fraction of output in order to maximize their dividend path. Adding low adjustment costs allows us to accurately match the observed volatility of residential investment. While the volatility of non-residential investment is slightly underestimated, the model is still able to correctly predict that non-residential investment is more than twice as volatile as output.

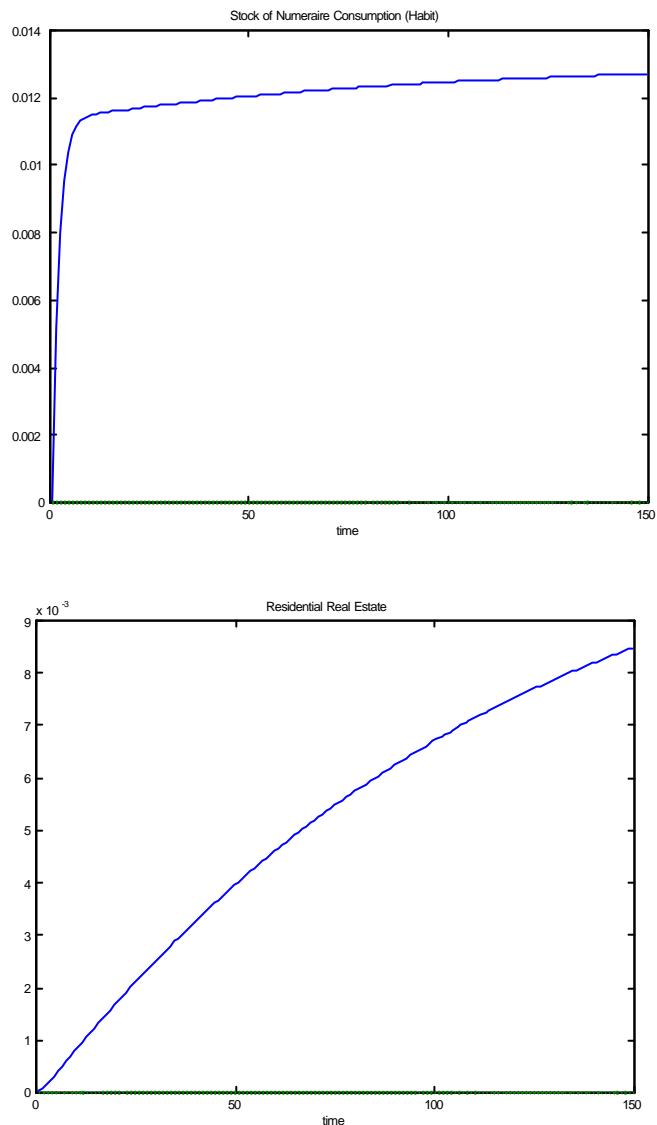
8.2 House Prices

As can be seen in table 3, the model is able to explain the fact that house prices are more volatile than output and the correlation between house prices and output is accurately reproduced. In contrast to most studies on the subject, it is possible to reproduce these two stylized facts in a model where the volatility of residential investment, the volatility of REIT price index and the risk premium on real estate returns are also accurately reproduced.

The mechanism driving the dynamics of house prices work as follows. Since house prices can be interpreted as the relative shadow price of the stock of residential real estate with respect to the stock of numeraire consumption (see section 11.5), house prices will depend on the parameters describing the accumulation of both stocks. In the case of the stock of consumption, a^c , being equal to 0 implies that the stock of habit completely depreciates from one period to the other, which therefore implies that numeraire consumption is a perishable good that cannot be stored more than one period. Second, a^h being equal to 0.995, indicates that while numeraire consumption completely depreciates, the rate of depreciation of real estate is very low.

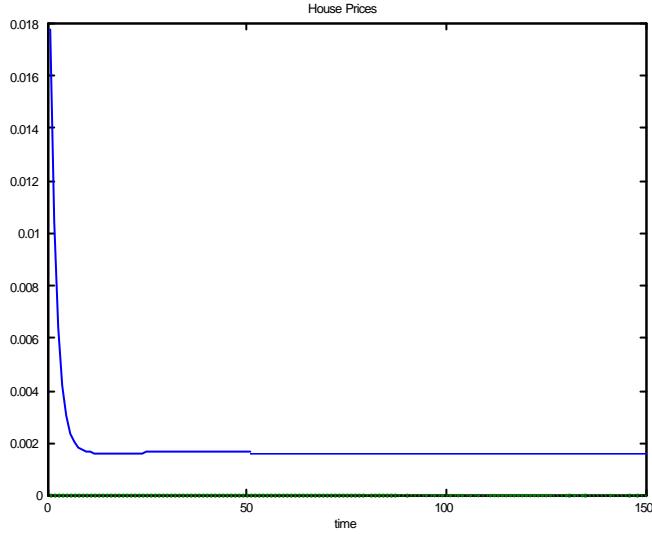
In the case of real estate, with b^h equal to 0.004, current consumption of residential real estate services only allows to increase the stock of residential real estate by a small amount. Increasing the stock of residential real estate is therefore costly and requires the agent to buy a lot of the composite real estate good. In contrast, the large value of b^c , which is set to 0.82, captures the fact that increasing the stock of the consumption good is not very costly.

As shown in the picture below, the stock of habit and residential real estate being both predetermined variables, these differences imply that, compared with numeraire consumption, the stock of residential real estate will respond more gradually to technology shocks.



As a result, the relative demand of residential real estate with respect to the stock of numeraire consumption will be negatively affected and the agent will start to accumulate more of the consumption good relatively to residential real estate in response to a productivity shock. A rise in the stock of habit having a positive impact on marginal utility of consumption, in response to a shock, the shadow price of numeraire consumption decreases to signal that this high level of habit harms the agent. In contrast, the sensitivity of the stock of residential real estate with respect to housing services being moderate, as the economy becomes more productive, since the stock of residential real estate increases much slower

than the stock of habit, the decline in the shadow price of residential real estate will be less pronounced. As a result, as shown in the following picture, house prices, as measures by the relative price of the stock of residential real estate with respect to numeraire consumption, respond positively.



To sum up, the key ingredients generating the high volatility of house prices are, firstly, the fact that numeraire consumption is a perishable good and so that it depreciates completely within the period and that, in contrast, the rate of depreciation of homes is very low; and second, that it is more costly to increase the stock of residential real estate, b^h being small compared to b^e . The fact that the stock of residential real estate depreciates slowly and that it is costly to increase it, leads the agent to accumulate numeraire consumption faster in response to a technology shock. The marginal utility of the stock of habit being negative, the relative shadow price of numeraire consumption rises to signal to the agent that the stock of numeraire consumption is too high compared to the stock of residential real estate. The rise in house prices is therefore due to the fact that optimality requires the agent to start accumulating more residential real estate compared to numeraire consumption.

8.3 Real Estate Returns and the Equity Premium

As far as modelling the non-residential sector is concerned, while in the model the numeraire sector is meant to represent the entire economy excluding real estate, a diversified global market index such as the MSCI equity index is chosen as a proxy. As for the real estate sector, the behavior of equity prices is compared with a real estate investment trust index, the NAREIT index, described earlier.

As far as the dynamics of equity prices is concerned, as shown in table 3, the model is able to account for the high observed volatility of both the

market index, \hat{p}_t^e , and the real estate investment trust price index, \hat{p}_t^r , and correctly predicts that equity prices in both sectors are more than seven times as volatile as output. As for returns, as in Jermann (1998), the combination of high capital adjustment costs and habit formation in consumption allows the model to explain simultaneously the high equity premium on both sectors returns and the low risk free rate using a coefficient of risk aversion which is in line with the empirical evidence.

As shown by the decomposition of the risk premiums (see the technical appendix, section 11.11), in both cases the term premium represents the main source of riskiness. This is due to the combination of high adjustment costs and habit formation. By causing, firstly, a reduction in the potential for intertemporal substitution and, secondly, an increase in the consumer's willingness to smooth consumption, habit formation and capital adjustment cost imply that a lower real interest rate is needed to induce the agent to save. The resulting negative impact on the real interest rate in response to an increase in productivity, which in turn will have a positive impact on prices, significantly contributes to increase the risk premiums on both real estate and equity returns, as this unfavorable correlation has to be compensated by a larger risk premium. The introduction of land, by implying a smaller share of capital in the production function and thus that less capital is needed, when combined with habit formation and capital adjustment costs also helps to reinforce this latter effect.

In terms of the questions stated in section 3, the success of the model to account simultaneously for the equity premium and the risk premium on real estate returns, while allowing to explain investment dynamics, therefore stems from the fact that land, in addition to habit formation and capital adjustment costs, has been introduced. In response to a productivity shock, the quantity of land being fixed, a rise in output in the numeraire sector causes the marginal productivity of land to increase. Land being rented to the firms in the numeraire sector, this positive effect of land on dividends of the real estate firm allows to dampen the negative effect due to the highly procyclical response of residential investment and therefore helps to increase the payout uncertainty premium. As a result, introducing land in the analysis allows to increase the riskiness of dividends in the real estate sector and therefore contributes to explain why despite the fact that residential investment is considerably more volatile than non-residential investment, real estate returns and equity returns are almost identical.

As explained earlier, the impact of land on real estate returns operates mainly through the term premium by increasing the sensitivity of prices to interest rates variations. To quantify this impact, setting the share of land to 0 would lead the risk premium on real estate returns to drop from 5.65% to 3.43%.

9 Conclusion

The main contribution of this paper is to develop an extended version of the neoclassical model that allows to study the link between real estate returns, house prices and the business cycle within the same framework. Compared to the existing literature, the advantage of this approach is that it allows to reproduce a large set of business cycle as well as asset market facts using only one single aggregate technology shock.

In terms of the objectives stated in the introduction, it has been shown that introducing habit formation and residential real estate in the utility function provides an explanation for the puzzling volatility of house prices. The model is also able to accurately reproduce the correlation between house prices and output. The key ingredients are, firstly, the fact that the consumption good depreciates very fast and that, in contrast, the rate of depreciation of homes is very low and secondly, that it takes time to increase the stock of residential real estate. In contrast to most studies on the subject, it is possible to reproduce this volatility and this correlation in a model where, in addition to the standard stylized facts, the volatility of residential investment, the volatility of real estate investment trust price index, the risk premium on real estate returns, the risk free rate and the equity premium are also accurately reproduced.

As far as asset pricing implications are concerned, it has firstly been shown that the model is able to account simultaneously for the puzzling equity premium [see Mehra and Prescott (1985)] and for the risk premium on real estate returns. Secondly, in terms of the questions stated in section 3, it has been shown that introducing land in the analysis allows to increase the riskiness of dividends in the real estate sector and therefore contributes to explain why despite the fact that residential investment is considerably more volatile than non-residential investment, real estate returns and equity returns are almost identical. Introducing land in the production function of the non-residential sector also helps to explain the high observed risk premiums by increasing the sensitivity of equity prices in both sectors to interest rate variations.

A main direction of future work will be to develop the implications of the analysis in terms of optimal portfolio allocation. Residential real estate being a major component of household wealth, general equilibrium predictions for the evolution of the share of housing over the business cycle could be exploited in tests of the model.

10 References

- Abel, A.B. (1996), "Risk Premiums and Term Premiums in General Equilibrium", unpublished paper, Wharton School of Economics, University of Pennsylvania
- Aghion P., Bacchetta P. and A. Banerjee (2003) "A Corporate Balance-sheet Approach to Currency Crises", Journal of Economic Theory, forthcoming
- Altig D., Christiano L., Eichenbaum M. and Linde J. (2003), "Technology Shock and Aggregate Fluctuations", manuscript, Northwestern University
- Aoki K., Proudman J. and Gertjan V. (2002) "House Prices, consumption and monetary policy: a financial accelerator approach", Bank of England's Working paper no.169
- Aoki K., Proudman J. and Vlieghe V. (2001) "House as a collateral: Has the Link between House Prices and Consumption Changed", mimeo, Bank of England
- Bansal R., Yaron A. (2002), "Risk for the long run: a potential resolution of asset pricing puzzles" Journal of Finance, forthcoming
- Baxter M. (1996), "Are consumer durables important for business cycles?" Review of Economics and Statistics LXXVIII:1 (February 1996), 147-155.
- Baxter M. and Crucini M (1993)., "Explaining Saving-Investment Correlations", The American Economic Review, vol. 83, no. 3 (Jun., 1993) 416-436
- Baxter M. and Jermann U., "Household Production and the Excess Sensitivity Consumption to Current Income", American Economic Review September (1999)
- Baxter and King (1993), "Fiscal Policy in General Equilibrium", American Economic Review 83: 315-334
- Benhabib J., Rogerson R. and Wright (1991), "Homework in Macroeconomics: Household Production and Aggregate Fluctuations", Journal of Political Economy 99, 1166-1187
- Benassy J-P. (1995), "Money and Wage Contracts in an Optimizing Model of the Business Cycle", Journal of Monetary Economics 35: 303-315
- Bernanke B and Gertler M. (1989) "Agency Costs, Net Worth and Business Fluctuations", American Economic Review, 79, March, 14-31
- Bernanke B, Gertler M and Gilchrist S. (2000), "The Financial Accelerator in a Quantitative Business Cycles Framework", Handbook of Macroeconomics, Volume 1C

- M. Boldrin, L.Christiano and J. Fisher (2001), "Habit Persistence, Asset Returns and the Business Cycle", American Economic Review (March 2001)
- Borio and Lowe (2002), "Asset Prices, Financial and Monetary Stability: Exploring the Nexus", BIS working paper no. 114
- Burnside, Eichenbaum and Fisher (2003), "Fiscal Shocks and Their Consequences", Working paper
- Burstein C., Eichenbaum M., Rebelo S. (1995), "Capacity Utilization and Return to Scale", NBER Macroeconomics Annual 1995, 67-110 MIT Press
- Campbell J.Y., "Asset Prices, Consumption and the Business Cycle", NBER working paper no. 6485
- Campbell J.Y. and J.H Cochrane, "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behaviour, unpublished paper, Harvard University and University of Chicago.
- Case K., Quigley J. and Shiller R. (2001), "Wealth Effects: The Stock Market versus The Housing Market", Working paper, Berkley E01-308
- Chari V., Kehoe P. and Ellen R. McGrattan (2003), "Business Cycles Accounting", Working Paper 625, Federal Reserve Bank of Minneapolis
- Christiano, Eichenbaum and Vigfusson (2003), "What Happens after a Technology Shock?", Working Paper
- Christiano L., Eichenbaum M. (1992) "Current real business cycle theories and aggregate labor market fluctuations", American Economic Review 82
- Cho, Man (1996), "House Price Dynamics: a survey of theoretical and empirical issues". Journal of Housing Research, 7: 145-72
- Cochrane J. (2001), "Asset Pricing", Princeton University Press
- Cooley T. and Prescott E. (1995), "Economic Growth and Business Cycles" in T. Cooley ed., Frontiers of Business Cycle Research, Princeton University Press, Princeton, 1-38
- Diaz-Gimenez J., Prescott E., Fitzgerald T. and Alvarez F. "Banking in Computable General Equilibrium Economies", Research Department Staff Report 153, Federal Reserve Bank of Minneapolis.
- Danthine J-P., Kurmann A. (2003), "Fair Wages in a New Keynesian Model of the Business Cycle", forthcoming in Review of Economic Dynamics
- Davis M., J. Heathcote (2003), "Housing and the Business Cycle", NBER Working Paper

- Fernandez-Villaverde J., and Krueger D.(2002), "Consumption and Saving over the Life-Cycle: How Important are Consumer Durables?", in Proceeding of the 2002 North American Summer Meetings of the Econometric Society.
- Flavin M., Yamashita T. (2002), "Owner-occupied housing and the composition of the household portfolio.", American Economic Review, March 345-62
- Fuhrer J. (2000), "Habit Formation in Consumption and Its Implications for Monetary Policy." American Economic Review. (September 2000).
- Gali (1999), "Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?", American Economic Review 89(1), 249-271
- Greenwood J., and Hercowitz Z. (1991), "The Allocation of Capital and Time over the Business Cycle," Journal of Political Economy 99, 1188-1214
- Greenwood J., Rogerson R. and Wright R. (1995), "Household Production in Real Business Cycle Theory" in T. Cooley ed., Frontiers of Business Cycle Research, Princeton University Press, Princeton, 157-174
- Hayashi F., "Tobin's marginal and average q: a neoclassical interpretation", Econometrica 50, 213-224
- Hoesli M. (1995), "Investissement Immobilier et Diversification de Portefeuille", Ecomomica, collection gestion.
- IMF (2002), "World Economic Outlook", September 2002
- Iacoviello (2003)," House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle", working paper
- Jermann U. (1994), Economic fluctuations and asset returns. Phd dissertation, University of Geneva
- Jermann U. (1998), Asset pricing in production economies, Journal of Monetary Economics 41
- Jin Y., Zeng Z. (2003), "Residential Investment and House Prices in a Multi-sector Monetary Business Cycle Model", Journal of Housing Economics 13 (2004) 268-286
- Kan K., S. Kwong, C. Leung, (2000), "The Dynamics and Volatility of Commercial and Residential Property Prices: Theory and Evidence", Journal of Real Estate Finance and Economics 2000
- King R., Plosser C., Rebelo .S (1989), Production growth and business cycles. Journal of Monetary Economics 21

- King R., Plosser C., Rebelo S (2002), "Production growth and business cycles: Technical Appendix" Journal of Computational Economics, October 2002
- King R., Rebelo S. (2000) "Resuscitating Real Business Cycles", Handbook of Macroeconomics
- Kocherlakota N. (1996) "The Equity Premium: It's Still A Puzzle", Journal of Economic Literature, 34: 42-71
- Kydland F. and Prescott E.(1988), "The Workweek of Capital and its Cyclical Implications", Journal of Monetary Policy (1988).
- Kydland F. and Prescott E.(1990) , "Business Cycles: Real Facts and a Monetary Myth" Federal Bank of Minneapolis, quarterly review 14: 3-18
- Leung C. (2004), "Macroeconomics and housing: a review of the literature" Journal of Housing Economics, vol. 13 issue 4.
- Long J. B. and Plosser C. (1983), "Real Business Cycles", Journal of Political Economy 91 (1), 39-69
- McCue T., Kling J.(1994), "Real Estate Returns and the Macroeconomy: Some Empirical Evidence from Real Estate Investment Trust". Journal of Real Estate Research, 1994, vol. 9, issue 3, pages 277-288
- Mehra R., Prescott E. (1985), The equity premium: a puzzle, Journal of Monetary Economics 15
- Normandin, M., St-Amour P., «Substitution, Risk Aversion, Taste Shocks and Equity Premiums», Journal of Applied Econometrics, 13, 265-281, 1998.
- Piazzesi M., Schneider M., Tuzel S. (2003), "Housing, Consumption and Asset Pricing", working paper UCLA Anderson School of Economics Department
- Platania J. and Schlagenauf D. (2000), " Housing and Asset Holding in a Dynamic General Equilibrium Model", Working Paper, Florida State University
- Pommeret Aude (2002), "Business Cycles", Lectures on Macroeconomics, University of Lausanne
- Thalmann, Philippe, and Milad Zarin-Nejadan (2003), Construction and Real Estate Dynamics, Palgrave Mcmillan pour Applied Econometrics Association, Basingstoke, UK, 2003
- Ortalo-Magne F. and S. Rady (2001), "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints", CEPR Working Paper 3015
- Ramey V. and Shapiro M. D. (1998), "Costly Capital Reallocation and the Effects of Government Spending", Carnegie Rochester Conference Series on Public Policy, June 1998

11 Technical Appendix

11.1 The Transformed Model

Following Burside C., Eichenbaum M., Rebelo S. (1995) this more general version of the model allows for varying rate of utilization in both sector, τ^e and τ^r denoting respectively the rate of utilization of the capital stock and of residential structures.

The representative agent maximizes expected lifetime utility given by:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_t(C_t, X_t^h, H_t^r, X_t^c) \right\}$$

where:

$$U_t(C_t, X_t^h, H_t^r, X_t^c) = \frac{1}{1-\sigma} [(C_t - m^c X_t^c)^\kappa (H_t^r - m^h X_t^h)^{1-\kappa}]^{1-\sigma}$$

and the production functions in both sectors are given by:

$$Y_t^e = A_t (\tau_t^e K_t^e)^\xi (\Gamma_t N_t^e)^\alpha H_t^{c(1-\alpha-\xi)} \quad (28)$$

and:

$$Y_t^r = A_t (\tau_t^r K_t^r)^\rho (\Gamma_t N_t^r)^{1-\rho} \quad (29)$$

To ensure steady state growth, technical progress (Γ_t) is introduced and is assumed to take a labor augmenting form [see King, Plosser and Rebelo (2002)]. The deterministic component of growth is given by:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma \quad (30)$$

To derive the stationary model, all variables have thus to be detrended and written in terms of variables that are constant in the steady state: $y_t^e = Y_t^e / \Gamma_t$, $y_t^h = Y_t^h / \Gamma_t$, $h_t^r = H_t^r / \Gamma_t$, $c_t = C_t / \Gamma_t$, $i_t^e = I_t^e / \Gamma_t$, $i_t^{nr} = I_t^{nr} / \Gamma_t$, $k_t^e = K_t^e / \Gamma_t$, $k_t^h = K_t^h / \Gamma_t$, $p_t^{hr} = P_t^{hr} / \Gamma_t$, $p_t^e = P_t^e / \Gamma_t$, $p_t^r = P_t^r / \Gamma_t$, $x_t^c = X_t^c / \Gamma_t$, $x_t^h = X_t^h / \Gamma_t$.

The problem of the consumer can be written in terms of stationary variables as:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\Gamma_0 \gamma^t)^{1-\sigma} \frac{1}{1-\sigma} [(c_t - m^c x_t^c)^\kappa (h_t^r - m^h x_t^h)^{1-\kappa}]^{1-\sigma} \right\} \quad (31)$$

Similarly, the constraints can be defined in stationary terms as:

$$W_t^h N_t^h + W_t^e N_t^e + S_t^e (p_t^e + d_t^e) + S_t^r (p_t^r + d_t^r) = c_t + z_t^r h_t^r + p_t^e S_{t+1}^e + p_t^r S_{t+1}^r \quad (32)$$

$$\gamma x_{t+1}^c = a^c x_t^c + b^c c_t \quad (33)$$

$$\gamma x_{t+1}^h = a^h x_t^h + b^h h_t^r \quad (34)$$

Defining $\beta^* = \beta\gamma^{1-\sigma}$ and using the fact that $\Gamma_t = \Gamma_0\gamma^t$ allows to put the problem into the form given in the text.

11.2 The Problem of the consumer

The Lagrangian for the problem of the representative consumer is given by :

$$\begin{aligned} L = & E_o \left\{ \sum_{t=0}^{\infty} \beta^{*t} \left[\frac{(c_t - m^c x_t^c)^\kappa (h_t^r - m^h x_t^h)^{1-\kappa}}{1-\sigma} \right]^{1-\sigma} \right. \\ & + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t [S_t^e(p_t^e + d_t^e) + S_t^h(p_t^h + d_t^h) - c_t - z_t^r h_t^r - p_t^e S_{t+1}^e - p_t^r S_{t+1}^r + W_t^r N_t^r + W_t^e N_t^e] \\ & \left. + \sum_{t=0}^{\infty} \beta^{*t} \phi_t^c [a^c x_t^c + b^c c_t - \gamma x_{t+1}^c] + \sum_{t=0}^{\infty} \beta^{*t} \phi_t^h [a^h x_t^h + b^h h_t^r - \gamma x_{t+1}^h] \right\} \end{aligned}$$

11.2.1 The First-order Conditions

c_t :

$$\kappa(c_t - m^c x_t^c)^{\kappa(1-\sigma)-1} (h_t^r - m^h x_t^h)^{(1-\kappa)(1-\sigma)} + b^c \phi_t^c = \lambda_t \quad (35)$$

h_t^r :

$$(1-\kappa)(c_t - m^c x_t^c)^{\kappa(1-\sigma)} (h_t^r - m^h x_t^h)^{(1-\kappa)(1-\sigma)-1} + b^h \phi_t^h = \lambda_t z_t^r \quad (36)$$

S_{t+1}^e :

$$\lambda_t p_t^e = \beta^* E_t \lambda_{t+1} [d_{t+1}^e + p_{t+1}^e] \quad (37)$$

S_{t+1}^h :

$$\lambda_t p_t^r = \beta^* E_t \lambda_{t+1} [d_{t+1}^r + p_{t+1}^r] \quad (38)$$

x_{t+1}^c :

$$\phi_t^c = \tilde{\beta}^* a^c E_t \phi_{t+1}^c - \tilde{\beta}^* m^c \kappa E_t (c_{t+1} - m^c x_{t+1}^c)^{\kappa(1-\sigma)-1} (h_{t+1}^r - m^h x_{t+1}^h)^{(1-\kappa)(1-\sigma)} \quad (39)$$

x_{t+1}^h :

$$\phi_t^h = \tilde{\beta}^* a^h E_t \phi_{t+1}^h + \tilde{\beta}^* m^h (1-\kappa) E_t (c_{t+1} - m^c x_{t+1}^c)^{\kappa(1-\sigma)} (h_{t+1}^r - m^h x_{t+1}^h)^{(1-\kappa)(1-\sigma)-1} \quad (40)$$

11.3 The Problem of the Real Estate Service Producer

The problem of the real estate good producer is similar to the problem of the numeraire good producer. The use of labor, N_t^r , and residential structures, k_t^r , is required to produce the composite real estate good and the firm is endowed with a fixed amount of land that is rented to the numeraire good producer at the rental price z_t^c . The dividend of the firm producing the real estate good is given by:

$$d_t^r = z_t^r y_t^r + z_t^c h_t^c - z_t^r i_t^r - W_t^r N_t^r \quad (41)$$

where z_t^r denotes the price of output produced in the real estate sector and the price of the real estate investment good. The production function is given by:

$$y_t^r = A_t (\tau_t^r k_t^r)^\rho N_t^{r(1-\rho)} \quad (42)$$

The problem of the real estate service producer can be described by the following program:

$$\begin{aligned} & \text{Max} \sum_{t=0}^{\infty} \beta^{*t} \lambda_t d_t^r \\ & \text{s.t. } d_t^r = z_t^r y_t^r + z_t^c h_t^c - z_t^r i_t^r - W_t^r N_t^r \end{aligned} \quad (43)$$

$$[1 - \delta^r(\tau_t^r)] k_t^r + \Phi^r(\frac{i_t^r}{k_t^r}) k_t^r = \gamma k_{t+1}^r \quad (44)$$

where the subscript r denotes variables used to describe the real estate sector
The Lagrangian associated with this problem is:

$$\begin{aligned} L = & E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \lambda_t z_t^r \left[A_t (\tau_t k_t)^{\rho} N_t^{1-\rho} + \frac{z_t^c}{z_t^r} h_t^c - i_t^r - \frac{W_t^r}{z_t^r} N_t^r \right] \right. \\ & \left. \sum_{t=0}^{\infty} \beta^{*t} \mu_t \left[[1 - \delta^r(\tau_t^r)] k_t^r + \Phi^r(\frac{i_t^r}{k_t^r}) k_t^r - \gamma k_{t+1}^r \right] \right\} \end{aligned} \quad (45)$$

where z_t^r denotes the price of real estate services.

11.3.1 The First-order Conditions

$$i_t^r :$$

$$\lambda_t z_t^r = \mu_t \Phi^r'(\frac{i_t^r}{k_t^r}) \quad (46)$$

$$N_t^r :$$

$$W_t^r = z_t^r (1 - \rho) \frac{y_t^r}{N_t^r} \quad (47)$$

k_{t+1}^r :

$$\mu_t = \tilde{\beta}^* E_t \lambda_{t+1} z_{t+1}^r \rho \frac{y_{t+1}^r}{k_{t+1}^r} + \tilde{\beta}^* E_t \mu_{t+1} \left[(1 - \delta^r(\tau_{t+1}^r)) - \Phi^r' \left(\frac{i_{t+1}^r}{k_{t+1}^r} \right) \frac{i_{t+1}^r}{k_{t+1}^r} + \Phi^r \left(\frac{i_{t+1}^r}{k_{t+1}^r} \right) \right] \quad (48)$$

τ_t^r :

$$\lambda_t z_t^r \rho \frac{y_t^r}{\tau_t^r} = \delta^r'(\tau_t^r) k_t^r \mu_t \quad (49)$$

μ_t :

$$[1 - \delta^r(\tau_t^r)] k_t^r + \Phi^r \left(\frac{i_t^r}{k_t^r} \right) k_t^r = \gamma k_{t+1}^r \quad (50)$$

11.4 The Problem of the Numeraire Consumption Producer

Managers maximize the value of the firm to its owners, the representative agent, which is equal the present discounted value of all current and expected cash flows d_t^e :

$$E_0 \sum_{t=0}^{\infty} \beta^{*t} \lambda_t d_t^e$$

with:

$$d_t^e = y_t^e - W_t^e N_t^e - z_t^c h_t^c - i_t^e \quad (51)$$

and where $\beta^{*t} \lambda_t$ is the marginal rate of substitution of the owner. y_t^e denotes output, W_t^e the wage rate paid to the workers and z_t^c the rental cost of land. The firm's capital stock obeys an intertemporal accumulation equation with adjustment cost and varying rate of capital utilization:

$$[1 - \delta^e(\tau_t^e)] k_t^e + \Phi^e \left(\frac{i_t^e}{k_t^e} \right) k_t^e = \gamma k_{t+1}^e \quad (52)$$

As in Baxter and Crucini (1993), the parameters of the capital adjustment costs function $\Phi^e \left(\frac{i_t^e}{k_t^e} \right)$ are set so that the model with adjustments costs has the same steady state as the model without adjustments costs and it is assumed that near the steady state point: $\Phi^e > 0$, $\Phi'^e > 0$ and $\Phi''^e < 0$. This captures the idea that increasing the capital stock rapidly is more costly than changing it slowly. The depreciation rate of capital $\delta^e(\tau_t^e)$ is positively affected by the rate of capital utilization τ_t^e , which captures the idea that when capital is used more intensively, it depreciates more rapidly [see Burnside, Eichenbaum and Rebelo (1995), King and Rebelo (2000), Kydland and Prescott (1988)]. The numeraire sector require the use of labor, N_t^e , capital, k_t^e , and land, h_t^e . The good is produced via Cobb-Douglas production function:

The Lagrangian associated with this problem is:

$$\begin{aligned}
L &= E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \lambda_t \left[A_t (\tau_t^e k_t^e)^\xi N_t^\alpha h_t^{c(1-\alpha-\xi)} - W_t^e N_t^e - z_t^c h_t^c - i_t^e \right] \right. \\
&\quad \left. \sum_{t=0}^{\infty} \beta^{*t} \varphi_t \left[[1 - \delta^e(\tau_t^e)] k_t^e + \Phi^e \left(\frac{i_t^e}{k_t^e} \right) k_t^e - \gamma k_{t+1}^e \right] \right\}
\end{aligned} \tag{53}$$

11.4.1 The First-order Conditions

i_t^e :

$$\lambda_t = \varphi_t \Phi^e' \left(\frac{i_t^e}{k_t^e} \right) \tag{54}$$

N_t^e :

$$W_t^e = \alpha \frac{y_t^e}{N_t^e} \tag{55}$$

k_{t+1}^e :

$$\varphi_t = \tilde{\beta}^* E_t \lambda_{t+1} \xi \frac{y_{t+1}^e}{k_{t+1}^e} + \tilde{\beta}^* E_t \varphi_{t+1} \left[(1 - \delta^e(\tau_{t+1}^e)) - \Phi^e' \left(\frac{i_{t+1}^e}{k_{t+1}^e} \right) \frac{i_{t+1}^e}{k_{t+1}^e} + \Phi^e \left(\frac{i_{t+1}^e}{k_{t+1}^e} \right) \right] \tag{56}$$

τ_t^e :

$$\lambda_t \xi \frac{y_t^e}{k_t^e} = \delta^e'(\tau_t^e) k_t^e \varphi_t \tag{57}$$

φ_t :

$$[1 - \delta^e(\tau_t^e)] k_t^e + \Phi^e \left(\frac{i_t^e}{k_t^e} \right) k_t^e = \gamma k_{t+1}^e \tag{58}$$

11.5 The Problem of the Social Planner

Starting from (3) and abstracting from the dynamics of capital, substituting out dividends using (13) and (16); and using the market clearing conditions (18) and (19), the budget constraint can be expressed as:

$$z_t^r y_t^h + y_t^e = c_t + i_t^e + z_t^r h_t^r + z_t^r i_t^r \tag{59}$$

where z_t^r denotes the relative price of the output good in the real estate sector and where the price of the output good in the non-residential sector has been normalized to one. Similarly, the constraint describing the evolution of the stock of habit and of the stock of residential real estate corresponding to this problem is:

$$\gamma p_t^h x_{t+1}^h + \gamma x_{t+1}^c = b^h p_t^h h_t + a^h p_t^h x_t^h + a^c x_t^c + b^c c_t \tag{60}$$

where p_t^h is the relative price of the stock of residential real estate in terms of the stock of numeraire consumption. The competitive equilibrium can thus be equivalently stated as the following centralized problem:

$$\begin{aligned}
L = & E_o \left\{ \sum_{t=0}^{\infty} \beta^{*t} [u_t(c_t, x_t^h, h_t^r, x_t^c)]^{1-\sigma} \right. \\
& + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t [z_t^r y_t^h + y_t^e - c_t - i_t^e - z_t^r h_t^r - z_t^r i_t^r] \\
& \left. \sum_{t=0}^{\infty} \beta^{*t} \phi_t [b^h p_t^h h_t + a^h p_t^h x_t^h + a^c x_t^c + b^c c_t - \gamma p_t^h x_{t+1}^h - \gamma x_{t+1}^c] \right\}
\end{aligned} \tag{61}$$

which can be rewritten as:

$$\begin{aligned}
L = & E_o \left\{ \sum_{t=0}^{\infty} \beta^{*t} [u_t(c_t, x_t^h, h_t^r, x_t^c)]^{1-\sigma} \right. \\
& + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t [y_t^e - c_t - i_t^e] + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t z_t^r [y_t^h - h_t^r - i_t^r] \\
& \left. \sum_{t=0}^{\infty} \beta^{*t} \phi_t p_t^h [b^h h_t + a^h x_t^h - \gamma x_{t+1}^h] + \sum_{t=0}^{\infty} \beta^{*t} \phi_t [a^c x_t^c + b^c c_t - \gamma x_{t+1}^c] \right\}
\end{aligned} \tag{62}$$

where λ_t , $\lambda_t z_t^r$, $\phi_t p_t^h$ and ϕ_t are the Lagrange multipliers attached to the four constraints. The first-order condition with respect to x_{t+1}^h describing the dynamics of house prices is:

$$p_t^h \phi_t = \tilde{\beta}^* a^h E_t p_{t+1}^h \phi_{t+1} + \tilde{\beta}^* E_t \frac{\partial u_{t+1}(c_{t+1}, x_{t+1}^h, h_{t+1}^r, x_{t+1}^c)}{\partial x_{t+1}} \tag{63}$$

solving forward, house prices can be expressed as¹⁸:

$$p_t^h = \tilde{\beta}^* E_t \sum_{k=0}^{\infty} \left(\tilde{\beta}^* a^h \right)^k \left(\frac{u'_{x_{t+k+1}^h}}{\phi_t} \right) \tag{64}$$

which is equivalent to the expression given in the text¹⁹:

$$p_t^h = \frac{\phi_t^h}{\phi_t^c}$$

11.6 The Adjustment Cost Function

Following Jermann (1998), we choose the following parametric form:

$$\phi\left(\frac{i}{k}\right) = \frac{b}{1-c} \left(\frac{i}{k}\right)^{1-c} + d \tag{65}$$

¹⁸ where $u'_{x_t^h}$ denotes the marginal utility of residential real estate.

¹⁹ The equivalence between the two definitions has been checked numerically.

As in Baxter and Crucini (1993), the parameters are set so that the model with adjustment costs has the same steady state as the model without adjustment cost. This implies that:

$$\begin{aligned}\phi\left(\frac{i}{k}\right) &= \gamma - (1 - \delta) \\ \phi'\left(\frac{i}{k}\right) &= 1 \\ \phi''\left(\frac{i}{k}\right) &= -\frac{c}{\gamma - (1 - \delta)}\end{aligned}\quad (66)$$

11.7 Steady State and Linearization

The model can be linearized around a deterministic steady state. We now show how the first-order condition with respect to consumption can be linearized using the steady state restriction implied by the model.

$$\kappa(c_t - m^c x_t^c)^{\kappa(1-\sigma)-1} (h_t^r - m^h x_t^h)^{(1-\kappa)(1-\sigma)} + b^c \phi_t^c = \lambda_t \quad (67)$$

Using the fact that (4) and (5) imply that in the steady state:

$$x^c = \eta^c c \quad (68)$$

$$x^h = \eta^h h^r \quad (69)$$

where:

$$\eta^c = \frac{b^c}{\gamma - \alpha^c} \quad (70)$$

and:

$$\eta^h = \frac{b^h}{\gamma - \alpha^h} \quad (71)$$

Then (39) in the steady state can be used to derive the following expression for the steady state value of ϕ^c :

$$\phi^c = \frac{m^c \kappa \beta^* (1 - m^c \eta^c)^{\kappa(1-\sigma)-1} (1 - m^h \eta^h)^{(1-\kappa)(1-\sigma)} c^{\kappa(1-\sigma)-1} h^r (1-\kappa)(1-\sigma)}{\beta^* a^c - \gamma} \quad (72)$$

Next, total differentiation of (35) gives the following expression:

$$\begin{aligned}&\kappa[\kappa(1 - \sigma) - 1](1 - m^c \eta^c)^{\kappa(1-\sigma)-2} c^{\kappa(1-\sigma)-2} (1 - m^h \eta^h)^{(1-\kappa)(1-\sigma)} h^r (1-\kappa)(1-\sigma) dc_t \\ &+ [\kappa(1 - \sigma)(1 - \kappa)](1 - m^c \eta^c)^{\kappa(1-\sigma)-1} c^{\kappa(1-\sigma)-1} (1 - m^h \eta^h)^{(1-\kappa)(1-\sigma)-1} h^r (1-\kappa)(1-\sigma) dh_t \\ &- \kappa[\kappa(1 - \sigma) - 1]m^c(1 - m^c \eta^c)^{\kappa(1-\sigma)-2} c^{\kappa(1-\sigma)-2} (1 - m^h \eta^h)^{(1-\kappa)(1-\sigma)} h^r (1-\kappa)(1-\sigma) dx_t^c \\ &- [\kappa(1 - \sigma)(1 - \kappa)]m^h(1 - m^c \eta^c)^{\kappa(1-\sigma)-1} c^{\kappa(1-\sigma)-1} (1 - m^h \eta^h)^{(1-\kappa)(1-\sigma)-1} h^r (1-\kappa)(1-\sigma) dx_t^h \\ &+ b^c d\phi_t^c \\ &= d\lambda_t\end{aligned}\quad (73)$$

since in the steady state we have that:

$$\kappa(1-m^c\eta^c)^{\kappa(1-\sigma)-1}c^{\kappa(1-\sigma)-1}(1-m^h\eta^h)^{(1-\kappa)(1-\sigma)}h^r(1-\kappa)(1-\sigma)+b^c\phi^c=\lambda \quad (74)$$

dividing (72) by (73) and using the expression computed earlier for ϕ^c enable us to derive the following log-linearized condition where $\hat{x}_t = \frac{x_t - \bar{x}}{x}$ denotes variables expressed in percentage deviation from steady state:

$$\begin{aligned} & [\kappa(1-\sigma)-1](1-m^c\eta^c)^{-1}\Upsilon_c^{-1}\hat{c}_t + [(1-\sigma)(1-\kappa)](1-m^h\eta^h)^{-1}\Upsilon_c^{-1}\hat{h}_t^r \\ & -[\kappa(1-\sigma)-1]m^c\eta^c(1-m^c\eta^c)^{-1}\Upsilon_c^{-1}\hat{x}_t^c - [(1-\sigma)(1-\kappa)]m^h\eta^h(1-m^h\eta^h)^{-1}\Upsilon_c^{-1}\hat{x}_t^h \\ & + \frac{m^cb^c\beta^*}{\beta^*a^c-\gamma}\Upsilon_c^{-1}\hat{\phi}_t = \hat{\lambda}_t \end{aligned} \quad (75)$$

where:

$$\Upsilon_c^{-1} = \left(1 + \frac{b^cm^c\beta^*}{\beta^*a^c-\gamma}\right)^{-1} \quad (76)$$

To simplify notation, we define a set a constant to express the above condition as:

$$C_c\hat{c}_t + C_h\hat{h}_t^r = C_{xc}\hat{x}_t^c + C_{xh}\hat{x}_t^h - C_\phi\hat{\phi}_t + \hat{\lambda}_t \quad (77)$$

11.8 The Dynamic System

Following the resolution method of King, Plosser and Rebelo (2002), the dynamic system is composed of two control variables c_t and h_t^r , four control state variables k_t^e , k_t^h , x_t^c and x_t^h , four co-state variables or shadow prices associated to each of the four control state variables λ_t , Λ_t^{20} , ϕ_t^h and ϕ_t^c and one exogenous state variable given by the random productivity shock A_t . The two remaining multipliers associated with the adjustment costs constraints in the problem of the two firms φ_t and μ_t [see (44) and (52)] can be substituted out using the two first-order conditions with respect to i_t^e and i_t^h , given by (46) and (54). The dynamic of the system is thus determined by a system of 8 equations and 8 unknowns k_t^e , k_t^h , x_t^c , x_t^h , λ_t , Λ_t , ϕ_t^c and ϕ_t^h . The solution of the system implies the computation of eight eigenvalues. Uniqueness of the solution and stability of the system require that the eigenvalues associated with the predetermined variables k_t^e , k_t^h , x_t^c and x_t^h are less than unity and that the eigenvalues associated with the non-predetermined variables λ_t , Λ_t , ϕ_t^c and ϕ_t^h are outside the unit circle.

- Equations linking the two control variables to the state variables²¹:

$$C_c\hat{c}_t + C_h\hat{h}_t^r = C_{xc}\hat{x}_t^c + C_{xh}\hat{x}_t^h - C_\phi\hat{\phi}_t + \hat{\lambda}_t \quad (78)$$

$$H_c\hat{c}_t + H_h\hat{h}_t^r = H_{xc}\hat{x}_t^c + H_{xh}\hat{x}_t^h - H_\mu\hat{\mu}_t + \hat{\Lambda}_t \quad (79)$$

²⁰where we define $z_t\lambda_t = \Lambda_t$

²¹where as in (77) the parameters premultiplying the variables are function of the structural parameters of the model.

- Equations linking the state variables to the co-state variables:

Real Estate sector:

$$-\widehat{\Lambda}_{t+1} + \widehat{\Lambda}_t - L_{k1}^{hr} \widehat{k}_{t+1}^h + L_k^{hr} \widehat{k}_t^h = L_{A1}^{hr} \widehat{A}_{t+1} + L_A^{hr} \widehat{A}_t + L_{h1}^{hr} \widehat{h}_{t+1}^{hr} - L_h^{hr} \widehat{h}_t^{hr} \quad (80)$$

$$\gamma \widehat{k}_{t+1}^{hr} + K_k^{hr} \widehat{k}_t^{hr} = K_A^{hr} \widehat{A}_t + K_{hr}^{hr} \widehat{h}_t^r \quad (81)$$

Numeraire Sector:

$$-\widehat{\lambda}_{t+1} + \widehat{\lambda}_t - L_{k^e 1}^e \widehat{k}_{t+1}^e + L_{k^e}^e \widehat{k}_t^e = L_{A1}^e \widehat{A}_{t+1} + L_A^e \widehat{A}_t + L_{c1}^e \widehat{c}_{t+1} - L_c^e \widehat{c}_t \quad (82)$$

$$\gamma \widehat{k}_{t+1}^e + K_k^e \widehat{k}_t^e = K_A^e \widehat{A}_t + K_c^e \widehat{h}_t^r \quad (83)$$

Consumer:

$$\phi_{xc1} \widehat{x}_{t+1}^c + \phi_{xh1} \widehat{x}_{t+1}^h + \phi_{\phi 1} \widehat{\phi}_{t+1} - \phi_\phi \widehat{\phi}_t = \phi_{c1} \widehat{c}_{t+1} + \phi_{h1} \widehat{h}_{t+1}^r \quad (84)$$

$$\gamma \widehat{x}_{t+1}^c - a^c \widehat{x}_t^c = (\gamma - a^c) \widehat{c}_t \quad (85)$$

$$\mu_{xc1} \widehat{x}_{t+1}^c + \mu_{xh1} \widehat{x}_{t+1}^h + \mu_{\mu 1} \widehat{\mu}_{t+1} - \mu_\mu \widehat{\mu}_t = \mu_{c1} \widehat{c}_{t+1} + \mu_{h1} \widehat{h}_{t+1}^r \quad (86)$$

$$\gamma \widehat{x}_{t+1}^h - a^h \widehat{x}_t^h = (\gamma - a^h) \widehat{h}_t^r \quad (87)$$

11.9 Dividends and Asset Prices

Dividends in the representative sector have been defined as:

$$d_t^e = y_t^e - W_t^e N_t^e - i_t^e - z_t^e h_t^c \quad (88)$$

which can be rewritten as:

$$d_t^e = \xi y_t^e - i_t^e \quad (89)$$

A log-linearized expression for dividend can be derived:

$$\widehat{d}_t^e = \xi \left(\frac{d_t^e}{y_t^e} \right)^{-1} \widehat{y}_t^e - \left(\frac{d_t^e}{i_t^e} \right)^{-1} \widehat{i}_t^e \quad (90)$$

where:

$$\frac{d_t^e}{y_t^e} = \xi - \frac{i_t^e}{y_t^e} \quad (91)$$

$$\frac{d^e}{i^e} = \xi \left(\frac{i^e}{y^e} \right)^{-1} - 1 \quad (92)$$

with:

$$\frac{i^e}{k^e} = \gamma - (1 - \delta^e) \quad (93)$$

$$\frac{i^e}{k^e} = \frac{i^e}{y^e} \frac{y^e}{k^e} \quad (94)$$

Dividends in the real estate sector defined in real terms are given by:

$$d_t^h = y_t^h - \frac{W_t^h}{z_t^r} N_t^h - i_t^h + \frac{z_t^c}{z_t^r} h_t^c \quad (95)$$

which can be rewritten as:

$$d_t^h = \rho y_t^h - i_t^h + \frac{z_t^c}{z_t^r} h_t^c \quad (96)$$

The log-linearized expression is:

$$\hat{d}_t^h = \rho \left(\frac{d^h}{y^h} \right)^{-1} \hat{y}_t^h - \left(\frac{d^h}{i^h} \right)^{-1} \hat{i}_t^h + \left(\frac{d^h}{z^c h^c} \right)^{-1} (\hat{z}_t^c - \hat{z}_t^r) \quad (97)$$

where:

$$\frac{d^h}{y^h} = \rho - \left(\frac{i^h}{y^h} \right)^{-1} + \frac{(1 - \alpha - \xi)}{z^r} \frac{y^e}{y^T} \left(\frac{y^h}{y^T} \right)^{-1} \quad (98)$$

$$\frac{d^h}{i^h} = \rho \left(\frac{i^h}{y^h} \right)^{-1} - 1 + \frac{(1 - \alpha - \xi)}{z^r} \left(\frac{i^h}{y^h} \right)^{-1} \frac{y^e}{y^T} \left(\frac{y^h}{y^T} \right)^{-1} \quad (99)$$

$$\frac{d^h}{z^c h^c} = \rho \left(\frac{y^e}{y^T} \right)^{-1} \left(\frac{y^h}{y^T} \right) \frac{z^r}{(1 - \alpha - \xi)} - \left(\frac{i^h}{y^h} \right) \left(\frac{y^e}{y^T} \right)^{-1} \left(\frac{y^h}{y^T} \right) \frac{z^r}{(1 - \alpha - \xi)} + 1 \quad (100)$$

$$z^r = \frac{(1 - \kappa)}{\kappa} \frac{(1 - m^c \eta^c)}{(1 - m^h \eta^h)} \frac{\Upsilon^h}{\Upsilon^c} \frac{c}{y^e} \frac{y^h}{h^r} \frac{y^e}{y^T} \left(\frac{y^h}{y^T} \right)^{-1} \quad (101)$$

and where $\frac{y^h}{y^T}$ denotes the share of the real estate sector in total output and $\frac{y^e}{y^T} = 1 - \frac{y^h}{y^T}$.
and with:

$$\frac{i^h}{k^h} = \gamma - (1 - \delta^h) \quad (102)$$

$$\frac{y^h}{k^h} = \frac{1 - \tilde{\beta}^* (1 - \delta^h)}{\rho \tilde{\beta}^*} \quad (103)$$

$$\frac{i^h}{k^h} = \frac{i^h}{y^h} \frac{y^h}{k^h} \quad (104)$$

11.10 The Risk Premium

Following Jermann (1998), expected returns can be defined as:

$$E_t (R_{t,t+1} [\{D_{t+k}\}_{k=1}^{\infty}]) = R_{t,t+1} [1_{t+1}] \sum_{k=1}^{\infty} w_t [D_{t+k}] \exp(\eta_h(k) + \eta_p(k)) \quad (105)$$

The solution of the model can be put into the state space form and represented by the following system:

$$S_t = MS_{t-1} + \varepsilon_t \quad (106)$$

$$Z_t = \Phi S_t \quad (107)$$

where the state is given by:

$$S'_t = [\hat{k}_t^e \hat{k}_t^h \hat{x}_t^c \hat{x}_t^h \hat{A}_t] \quad (108)$$

where Z_t denotes a vector of variables depending on the state:

$$Z'_t = [\hat{d}_t^e \hat{d}_t^h \hat{\lambda}_t] \quad (109)$$

and where ε_t is a vector of random disturbances that are $NIID(0, \sigma^2)$

Using this solution method, dividends and marginal utility can be expressed in terms of the state vector as:

$$\hat{d}_t^e = l_{d^e} Z_t = l_{d^e} \Phi S_t \quad (110)$$

$$\hat{d}_t^h = l_{d^h} Z_t = l_{d^h} \Phi S_t \quad (111)$$

$$\hat{\lambda}_t = l_{\lambda} Z_t = l_{\lambda} \Phi S_t \quad (112)$$

Defining the loading vectors as:

$$d_{d^e} = l_{d^e} \Phi \quad (113)$$

$$d_{d^h} = l_{d^h} \Phi \quad (114)$$

$$d_{\lambda} = l_{\lambda} \Phi \quad (115)$$

expected returns can be solved and, in the case of the representative sector, are given by:

$$E_t (R_{t,t+1} [\{D_{t+k}\}_{k=1}^{\infty}]) =$$

$$\sum_{k=1}^{\infty} \left\{ \frac{\beta^{*k} \exp \left[d_{\lambda}(M^k - I)S_t + d_{d^e} M^k S_t + d^e + \frac{1}{2} \sum_{j=1}^k d_{\lambda} M^{k-j} \Sigma M^{k-j'} d'_{\lambda} \right]}{\sum_{k=1}^{\infty} \beta^{*k} \exp \left[d_{\lambda}(M^k - I)S_t + d_{d^e} M^k S_t + d^e + \frac{1}{2} \sum_{j=1}^k d_{\lambda} M^{k-j} \Sigma M^{k-j'} d'_{\lambda} \right]} \times \exp(\eta_h(k) + \eta_p(k)) \right\} \quad (116)$$

where:

$$\eta_h(k) = -d_{\lambda} \cdot \Sigma \cdot (M^{k-1} - I)' \cdot d'_{\lambda} \quad (117)$$

$$\eta_p(k) = -d_{\lambda} \cdot \Sigma \cdot M^{k-1'} \cdot d'_{d^e} \quad (118)$$

with:

$$\Sigma = E(\varepsilon_t \varepsilon_t') \quad (119)$$

and where I is an identity matrix and d^e denotes the steady state level of dividends²².

²²Since the model gives solutions in log deviation from steady state and that the computation of the risk premium require log variables, this adjustment in the term defining the weights is needed. The results are however independant of the value of d^e .

11.11 Decomposition of the Equity Premium and of the Real Estate Risk Premium

