Mathematical analysis - basic concepts

1. Derivative of scalar and vector functions with respect to a vector of variables.
2. Show that for column vectors $\mathbf{a}$ and β the following holds: $\frac{\partial \mathbf{a}'\beta}{\partial \beta} = \mathbf{a}$ i $\frac{\partial \mathbf{a}'\beta}{\partial \beta'} = \mathbf{a}'$
3. Show that for $\frac{\partial \mathbf{A}\beta}{\partial \beta'} = \mathbf{A}$ i $\frac{\partial \beta'\mathbf{A}}{\partial \beta} = \mathbf{A}$
4. Show that $\frac{\partial \beta'\mathbf{A}\beta}{\partial \beta'} = 2\mathbf{A}\beta$
5. What values should be assumed by the gradient of a continuous and differentiable function $f(\beta)$ at point β^*, so that β^* could be a maximum point.
6. What are the characteristics of second derivatives' matrix?
7. How can you recognise (basing on the characteristics of the second derivatives' matrix) whether an identified extremum is a maximum?

Problems

1. Find the gradient and the Hessian for $y = 2x_1^2 + 3x_2^2 + 5x_1x_2 - 4$. Find the extremum and identify whether this is a maximum or a minimum? Local or global?
2. Find the extremum of the function $y = x_1^2 + 4x_2^2 + x_1x_2 - 1$ and characterise it. Find this extremum subject to the constraint $x_2 - 2x_1 = 1$. Use the Lagrangean pasting the constraints directly to the objective function. Compare the extrema in both cases.
3. The following maxima have been identified: $g^* = \max_{x_1,x_2} g(x_1, x_2)$ i $g^{**} = \max_{x_1} g(x_1, 0)$. How are g^* i g^{**} related?
4. We have found the maxima with following constraints $g^* = \max_{x_1,x_2} g(\mathbf{x})$ s.t. $\mathbf{H}(\mathbf{x}) = 0$ and a maximum without constraints $g^{**} = \max_{x_1,x_2} g(\mathbf{x})$. What is the relation between g^* i g^{**}?
5. A maximum with the constraints has been found $g^* = \max_{x_1,x_2} g(\mathbf{x})$ s.t. $\mathbf{H}(\mathbf{x}) = 0$. During the procedure, it has turned out that i^{th} line of matrix $\mathbf{H}(\mathbf{x})$ is positive at maximum point. ($\mathbf{H}_i(\mathbf{x}^*) > 0$). What is the value for the Lagrangean for the i^{th} constraint in this problem?