The Dynamic Effects of Aggregate Demand and Supply Disturbances: Reply

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Marco Lippi and Lucrezia Reichlin (1993), in their criticism of our work, have uncovered a Pandora's Box of difficulties in interpreting VAR's. In this reply we give our evaluation of the significance of these difficulties; we outline what we take to be the key contributions and shortcomings of Lippi and Reichlin's note. At the end, we will conclude that the points that Lippi and Reichlin (hereafter LR) raise are relevant for macroeconometric practice in general, beyond their application to our work. Finally, we provide yet a new reason for macroeconometricians to take heed of LR's criticism; this reason lies in the nonfundamentalness that can be induced by the simple mechanics of manipulating cointegrated systems. Such a link between nonfundamentalness and cointegration has not, to our knowledge, been made explicit in the literature.

I. Nonfundamental Representation

LR construct *nonfundamental* representations from the VAR's that we estimated. Such representations can potentially alter our conclusions on the relative importance and the dynamics of the different kinds of disturbances that we identified. To clarify this, it is useful to recall some time-series facts—in a way different from that given in LR.

Fundamental representations are moving averages in disturbances where the disturbances can be recovered as (mean-square limits of) one-sided convergent distributed lags in observable variables. Nonfundamental representations are those where they cannot. The implications of this distinction are most easily understood by considering the univariate case; extension to multivariate models involves no new ideas.

Take the simplest possible example. Suppose that X is white noise, with disturbance ε:

$$X(t)=\varepsilon(t).$$

Notice that ε itself might be a distributed lag function of yet another white-noise disturbance:

$$\varepsilon(t)=\frac{1-\lambda_1 L}{1-\lambda_1^{-1}L}\eta_1(t)$$

for any λ_1 with modulus greater than 1, and η_1 white noise. (To see this, just calculate the covariogram of the right-hand side, and notice that it vanishes at all nonzero lags.) Such a moving-average representation,

$$X(t)=\frac{1-\lambda_1 L}{1-\lambda_1^{-1}L}\eta_1(t)$$

$$=\eta_1(t)+\left(1-\lambda_1^2\right)\sum_{j\geq 1}\lambda_1^{-j}\eta_1(t-j)$$

gives disturbances η_1 that are nonfundamental for observed X. When λ_1 is positive, then the dynamic response of X to η_1 is contemporaneously positive, after that negative, and subsequently it decays monotonically to zero. If, on the other hand, λ_1 is negative, then the response is positive for the first two periods, and after that it oscillates around zero, again decaying over time.

There might appear to be a distinctive pattern in the moving-average coefficients above: if the initial impact is positive, then negative coefficients follow; if the initial impact is negative, then positive coefficients follow. A balancing of negative and positive

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effects seems to occur so that the resulting dynamics remain those of white noise.

Such a distinctive pattern is, however, spurious; as the notation above suggests, we can further take η_1 to be a distributed lag function of yet another white noise,

$$\eta_1(t) = \frac{1-\lambda_2 L}{1-\lambda_2^{-1}L}\eta_2(t) \qquad |\lambda_2|>1$$

and even allow the λ coefficients to be complex by taking them in a conjugate pair:

$$\eta_1(t) = \frac{1-\lambda_2 L}{1-\lambda_2^{-1}L}\,\frac{1-\bar{\lambda}_2 L}{1-\bar{\lambda}_2^{-1}L}\eta_3(t) \qquad |\lambda_2|>1.$$

What do we learn from this? We realize that certain nonfundamental moving-average patterns turn out to be impossible for particular processes—for instance, a white-noise sequence cannot, except in its fundamental representation, have all nonnegative coefficients in its moving average. But, taking different convolutions of the factors above also establishes that almost any other pattern in the moving-average coefficients —hump-shaped or otherwise—is achievable, subject only to the constraint that the zeros of the numerator moving average are inside the unit circle.[1] The resulting moving-average representation,

$$X(t) = \prod_{j=1}^{J}\frac{1-\lambda_j L}{1-\lambda_j^{-1}L}\eta_J(t) = C_J(L)\eta_J(t)$$

always gives white noise, provided that $|\lambda_j|>1$. When, to begin, X is not white noise, the possibilities become richer, and a process can have moving-average representations that mix fundamental and nonfundamental parts.

Realizing this, we see that the LR criticism of our work is really independent of structural VAR interpretation in specific, and empirical VAR analysis in general. It is true that VAR models, as typically manipulated, recover only disturbances that are fundamental; but so does almost every other model used in analyzing time-series data. The problem that LR raise remains pertinent in even the most straightforward Box-Jenkins time-series analysis.[2]

Saying this, of course, does not absolve us and other VAR users from the responsibility of dealing with this difficulty. Some will just assume away the supposed subtlety (as we did in the first line of page 657 of our original article). That assuming away, however, becomes more incredible, and the supposed subtlety becomes less subtle, as examples accumulate showing the economic relevance of nonfundamental representations. In addition to LR, the published record shows such examples in the studies of Carl Futia (1981), Robert Townsend (1983), Quah (1990), and Hansen and Sargent (1991), among others. These studies have presented economic models where nonfundamentalness is likely to arise: a unifying thread running through these papers is the feature that economic agents observe information that the econometrician does not.

In those models, nonfundamentalness turns out to be the natural, and necessary, consequence of economic reasoning. A simple example is useful here to make the point concrete.[3] In the Friedman-Muth model of permanent income and consumption, permanent and transitory disturbances perturb income:

$$Y(t) = Y_1(t) + Y_0(t)$$

[1]It is useful here to note the difference between fundamental and invertible moving-average representations. Invertibility rules out, in addition, moving averages whose coefficients sum to zero (i.e., have a "unit root" in the moving average). Because such processes can be arbitrarily well approximated in mean square by processes with no such unit roots, those disturbances remain fundamental for the observed process. Thus, for example, while $\mathbf{X}(t) = (1-L)\varepsilon(t)$ is noninvertible, the disturbance ε remains fundamental for $\mathbf{X}$. This makes clear that the nonfundamentalness under discussion here is not simply a matter of overdifferencing.

[2]As LR emphasize, some of the points that they raise are also made (in a purely theoretical setting) by Lars Peter Hansen and Thomas Sargent (1991).

[3]This example is modified from Quah (1990 pp. 471–2).

with

$$\Delta Y_1(t) = \varepsilon_1(t)$$

$$Y_0(t) = \varepsilon_0(t).$$

(The disturbances ε_1 and ε_0 are uncorrelated white noises.) If consumption obeys the permanent-income hypothesis, as for example in Robert Hall (1978), then

$$\Delta C(t) = \varepsilon_1(t) + (1-\beta)\varepsilon_0(t)$$

where β (between 0 and 1) is the consumer's subjective discount factor. Putting these together we have

$$\begin{bmatrix} \Delta Y(t) \\ \Delta C(t) \end{bmatrix} = \begin{bmatrix} 1 & 1-L \\ 1 & 1-\beta \end{bmatrix} \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_0(t) \end{bmatrix}.$$

The determinant of the moving-average function is $(1-\beta)-(1-z) = z-\beta$ and thus vanishes at $z = \beta$, *inside* the unit circle. This moving-average representation for $(\Delta Y, \Delta C)$—the natural consequence of the permanent-income hypothesis—is thus nonfundamental.

As suggested by the LR reasoning, the permanent and transitory components in Y are then not recoverable from the joint history of Y and C. From the moving average of $(\Delta Y, \Delta C)'$ we see that the vector $(Y, C)'$ is not cointegrated. A VAR for $(\Delta Y, \Delta C)'$ is thus well specified; if one calculates such a VAR and then uses the identification scheme suggested in our original paper, one recovers uniquely:[4]

$$\begin{bmatrix} \Delta Y(t) \\ \Delta C(t) \end{bmatrix} = \phi(\beta)^{-1} \begin{bmatrix} 3-\beta-(1-\beta)^2 L & [\beta-(1-\beta)^2](1-L) \\ \phi(\beta) & -(1-\beta)\phi(\beta) \end{bmatrix} \times \begin{bmatrix} \nu_1(t) \\ \nu_0(t) \end{bmatrix}$$

[4]The unique representation given here arises from the informational restriction used in our original paper or, equivalently, the fundamentalness that LR are criticizing.

with $\phi(\beta) = 1 + (1-\beta^2)$. The first row of this certainly gives permanent and transitory components in Y, associated with ν_1 and ν_0, respectively. Note, however, that here the identified permanent disturbance ν_1 has first a positive impact on income Y, $(3-\beta)/\phi(\beta)$, and then a reduced effect, $[(3-\beta)-(1-\beta)^2]/\phi(\beta)$. By contrast, the true Friedman-Muth permanent disturbance ε_1 has only a one-time permanent effect.

To summarize this section, the first lesson is that nonfundamental representations need have no distinctive shape: from this we conclude that such representations cannot, on a priori grounds, be excluded from consideration. The opposite question—whether there is reason for considering nonfundamental representations in the first place—is answered positively, for different economic models, by the references above. The example we give also shows explicitly that nonfundamentalness can arise in a completely innocent, uncontrived way.

We now turn to LR's own argument for allowing nonfundamental representations.

II. The Lippi-Reichlin Criticism

In our view, criticism along the lines of LR's will significantly affect macroeconometric practice if the underlying ideas satisfy at least one of the following criteria:

(i) alternative nonfundamental representations display properties that are economically sensible, yet markedly different from those in fundamental representations;
(ii) an interesting economic hypothesis produces the nonfundamentalness;
(iii) practitioners face pervasive risk of inadvertently inducing nonfundamentalness in their interpretation of data.

Before we discuss it more fully below, notice here that (iii) differs from a researcher's simply not taking nonfundamentalness to be a relevant possibility; we mean to emphasize in (iii) that nonfundamentalness might actually be created in the process of manipulating a time-series model.

From the perspective of (i) we are struck by the similarity of LR's reported impulse responses, both across their different experiments and compared to the impulse responses in our original paper. It is true that the variance decompositions (equivalently, the impulse responses) change at short horizons, but the general pattern of impulse responses is quite close to what we obtained. We conclude that the resulting dynamic variance decompositions, while differing in details at particular horizons, must be surprisingly similar to ours.

This emphasizes the importance of theoretical propositions such as that which LR present, following their equation (14), in delineating just how different the empirical results could have turned out to be. That result shows that a researcher can always produce a nonfundamental representation which allocates to the hypothesized demand disturbance any relative importance from 0 to 1. That LR did not present such an example is (we conjecture) due to the unreasonable interpretations that would have to be given to it in the current demand–supply work.

By contrast, from the perspective of (ii) we find LR's reasoning to be unsatisfactory. Admittedly they intend the analysis in their section III to be no more than suggestive. Certain points in that section, however, do bear emphasis. Notice that the only reason nonfundamentalness arises in their model is through the moving-average representation for $\Delta\theta$. LR *assume* this representation to be nonfundamental. Why? Would any researcher—counterfactually, having observations on and interested in fitting a time-series model for $\Delta\theta$—have assumed this? We think not; but then why bring in that complication when, to begin, $\Delta\theta$ is unobservable?

We are also unconvinced that the learning-by-doing or knowledge-accumulation process (the S-shaped diffusion curve in LR) necessitates a nonfundamental moving average. Instead, we see that their normalized S-shaped diffusion curve, in another guise, is simply a cumulative distribution function; and for that, all that is needed is a moving-average representation whose coefficients are nonnegative: plenty of fundamental moving averages will do that nicely. There is no way to tell, apart from explicitly calculating the zeros, when a moving average having all nonnegative coefficients is fundamental and when it is nonfundamental. This inability to say anything concrete by purely analytical methods should not be confused with a necessity that the representation be nonfundamental.

Despite the criticisms we have made here, we acknowledge that we accede the point to LR. We, as have many others, simply ignored an important additional dimension in the identifying of VAR models. The entire point of structural VAR analysis is exactly identification of alternative disturbances: all possibilities need to be considered.

III. Cointegrated Models

In this section, we now propose to add a new reason to the list of how nonfundamental representations could arise: this takes us back to our point (iii) above. It turns out that the Beveridge-Nelson decomposition (Stephen Beveridge and Charles Nelson, 1981) potentially induces nonfundamentalness. This fact does not seem to be widely recognized; it will be embedded in our example below. More importantly, however, recall that Beveridge-Nelson-like calculations are used to derive common-trends representations in cointegrated time series (e.g., Robert Engle and Clive Granger, 1987). Thus it would seem that nonfundamentalness is a property that is especially relevant in cointegrated models. Again, this possibility has not been raised in the literature.

To make our assertion precise, let $\mathbf{X}$ be a vector of integrated series; let its first differences have the fundamental moving-average representation:

$$\Delta\mathbf{X}(t) = \mathbf{C}(L)\boldsymbol{\varepsilon}(t)$$

$$\det\mathbf{C}(z) \neq 0 \text{ for all } |z| < 1$$

$$\boldsymbol{\varepsilon} \text{ serially uncorrelated.}$$

As in our original article, a researcher is interested in the dynamic effects of ε on $\mathbf{X}$; again, some theoretical reasoning might suggest those to be of economic significance. Do standard procedures used in the analysis of cointegrated systems correctly recover ε, and thence those dynamic effects? We show below that for common-trends representations this is, in general, not guaranteed, thus extending the scope of LR's criticism in a new and fertile direction.

Following Engle and Granger (1987), we apply Beveridge-Nelson calculations to get the following representation:

$$(1) \qquad \mathbf{X}(t) = \mathbf{X}(0) + \mathbf{C}(1) \sum_{j=1}^{t} \boldsymbol{\varepsilon}(j) + \mathbf{C}^*(L)\boldsymbol{\varepsilon}(t) \qquad t \geq 1$$

where the coefficients in $\mathbf{C}^*$ are given by

$$C_j^* = -\sum_{k>j} C_k \qquad j \geq 0.$$

This equation decomposes $\mathbf{X}$ into so-called long-run and short-run dynamics; many cointegration representations derive from exactly such a decomposition.

In cointegration analysis most attention has focused on the properties of the matrix $\mathbf{C}(1)$. The dynamics in the stationary residual $\mathbf{C}^*(L)\boldsymbol{\varepsilon}$ are captured, in practice, by including sufficient lags of $\Delta\mathbf{X}$ in the empirical analysis. From the perspective of the issues discussed above, such a procedure would be unobjectionable if $\mathbf{C}^*$ were to be invertible.

It is easy to show that when $\mathbf{X}$ is I(1)—or, more correctly, when $d\mathbf{C}(z)/dz$ is full-rank at $z=1$—then $\mathbf{C}^*(1)$ is full-rank. Thus, $\det \mathbf{C}^*$ has no zeros at $z=1$. That fact alone, however, does not imply that $\mathbf{C}^*$ is invertible; for that, $\mathbf{C}^*(z)$ must be full-rank *everywhere* on $|z| \leq 1$.

A simple example suffices to show that no reasonable regularity conditions will guarantee this last property. Suppose that for $|\lambda_1|$, $|\lambda_2|$, and $|\rho|$ less than 1, with ρ different from zero, we have:

$$\mathbf{C}(z) = \begin{bmatrix} (1-\lambda_1 z)(1-\lambda_2 z) & (1-\lambda_1)(1-\lambda_2)(1-\rho) \\ 1 & 1-\rho z \end{bmatrix}$$

so that

$$\det \mathbf{C}(z) = (1-\lambda_1 z)(1-\lambda_2 z)(1-\rho z) - (1-\lambda_1)(1-\lambda_2)(1-\rho).$$

The function $\det \mathbf{C}$ clearly vanishes at $z=1$, and thus $\mathbf{X}$ above is cointegrated. Further, for ρ close to 1, $\det \mathbf{C}$ always has its remaining zeros close to λ_1^{-1} and λ_2^{-1}, and thus strictly outside the unit circle. For instance, if ρ is 0.95, λ_1 is 0.6, and λ_2 is -0.7, then the two zeros of $\det \mathbf{C}$, other than at unity, are at -1.417 and 1.708.

In the present example, the corresponding common-trends representation (1) has short-run dynamics given by

$$\mathbf{C}^*(z) = \begin{bmatrix} [(\lambda_1+\lambda_2)-\lambda_1\lambda_2] - \lambda_1\lambda_2 z & 0 \\ 0 & \rho \end{bmatrix}$$

so that

$$\det \mathbf{C}^*(z) = \rho\big([(\lambda_1+\lambda_2)-\lambda_1\lambda_2] - \lambda_1\lambda_2 z\big).$$

This determinantal function has only one zero, at $[(\lambda_1+\lambda_2)-\lambda_1\lambda_2]/\lambda_1\lambda_2$: there is no guarantee that this zero lies outside the unit circle. In fact, for the numerical example we gave above, this zero is at $-16/21$, well inside the unit circle. We conclude that $\mathbf{C}^*$ corresponds to a nonfundamental representation. In other words, for this model the common-trends representation has short-run dynamics $\mathbf{C}^*$ whose disturbances differ substantively from the original ones: those disturbances of interest are never recoverable by *any* orthogonalization of the short-run components' innovations from a common-trends model.

In this section we began with a model for which the disturbances of interest are fundamental. We have shown that one standard representation for cointegrated systems—the common-trends one—yields short-run dynamics that are intrinsically different from those of the original disturbances. Thus analyzing that representation does not obtain the disturbances or the dynamics of interest. By contrast, LR suggest that nonfundamental disturbances might be of interest. If so, standard methods—such as in our original article—are deficient, since those can only recover fundamental disturbances.

It is useful to remark here that the wealth of literature on cointegration has focused on statistical inference, rather than on the issue of identification of disturbances. For inference, it is immaterial whether or not a particular representation is fundamental. For identification, it is very material, as LR show.

IV. Conclusion

To summarize, we agree with the main thrust of LR's criticism, although we are critical of details in their implementation. The identification problem that they raise —the existence of nonfundamental representations—is an issue whenever a researcher wishes to give an economic interpretation to time series.

We have argued that nonfundamental representations are not absolutely necessary to capture many dynamic effects of interest. At the same time, neither can those representations be dismissed always and everywhere. Sometimes, they turn out to be precisely the phenomena of interest.

That constitutes the first part of our reply. For Section III, it is useful to relate what we have done here with the literature's progression, following LR. Our original article began by assuming a fundamental moving-average representation in aggregate demand and supply disturbances; our paper then showed how to recover those disturbances using a VAR. LR's note begins by assuming a nonfundamental representation and then criticizes our empirical work because our VAR, as we manipulated it, could not recover the nonfundamental disturbances. Up to this point, however, the question is a matter of choice in assumptions. By contrast, the discussion of Section III shows how a nonfundamental representation can naturally arise in cointegrated systems. Such systems are now routinely used in empirical work in macroeconomics. We have shown here that the force of LR's criticism applies, even more strongly, to such systems [see point (iii) of our Section II]. Unlike users of standard VAR's (such as in our application to demand and supply disturbances), users of common-trends models need to prove that the disturbances they uncover are the ones originally of interest: they do not have the freedom simply to make that assumption, as we did.

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