The Dynamic Effects of Aggregate Demand and Supply Disturbances

By Olivier Jean Blanchard and Danny Quah*

We interpret fluctuations in GNP and unemployment as due to two types of disturbances: disturbances that have a permanent effect on output and disturbances that do not. We interpret the first as supply disturbances, the second as demand disturbances. Demand disturbances have a hump-shaped mirror-image effect on output and unemployment. The effect of supply disturbances on output increases steadily over time, peaking after two years and reaching a plateau after five years.

It is now widely accepted that GNP is reasonably characterized as a unit root process: a positive innovation in GNP should lead one to revise upward one's forecast on GNP for all horizons. Following the influential work of Charles Nelson and Charles Plosser (1982), this statistical characterization has been recorded and refined by numerous authors including John Campbell and N. Gregory Mankiw (1987a), Peter Clark (1987, 1988), John Cochrane (1988), Francis Diebold and Glenn Rudebusch (1988), George Evans (1987), and Mark Watson (1986).

How should this finding affect one's views about macroeconomic fluctuations? Were there only one type of disturbance in the economy, then the implications of these findings would be straightforward. That disturbance would affect the economy in a way characterized by estimated univariate-moving average representations, such as those given by Campbell and Mankiw. The problem would simply be to find out what this disturbance was, and why its dynamic effects had the shape that they did. The way to proceed would be clear.

However, if GNP is affected by more than one type of disturbance, as is likely, the interpretation becomes more difficult. In that case, the univariate-moving average representation of output is some combination of the dynamic response of output to each of the disturbances. The work in Stephen Beveridge and Nelson (1981), Andrew Harvey (1985), and Watson (1986) can be viewed as early attempts to get at this issue.[1]

To proceed, given the possibility that output may be affected by more than one type of disturbance, one can impose a priori restrictions on the response of output to each of the disturbances, or one can exploit information from macroeconomic variables other than GNP. In addition to the work named above, Clark (1987) has also used the first approach. This paper adopts the second, and considers the joint behavior of output and unemployment. Campbell and Mankiw (1987b), Clark (1988), and Evans (1987) have also taken this approach. Our analysis differs mainly in its choice of identifying restric-

*Both authors are with the Economics Department, MIT, Cambridge MA 02139, and the NBER. We thank Stanley Fischer, Julio Rotemberg, Mark Watson for helpful discussions, and the NSF for financial assistance. We are also grateful for the comments of two anonymous referees and of participants at an NBER Economic Fluctuations meeting, and for the hospitality of the MIT Statistics Center.

[1]As will become clear, our work differs from these in that we wish to examine the dynamic effects of disturbances that have permanent effects; such issues cannot be addressed by studies that restrict the permanent component to be a random walk. In other work, one of us has characterized the effects of different parametric specifications (such as lag length restrictions, a rational form for the lag distribution) for the question of the relative importance of permanent and transitory components. See Quah (1988).

tions; as we shall argue, we find our restrictions more appealing than theirs.

Our approach is conceptually straightforward. We assume that there are two kinds of disturbances, each uncorrelated with the other, and that neither has a long-run effect on unemployment. We assume however that the first has a long-run effect on output while the second does not. These assumptions are sufficient to just identify the two types of disturbances, and their dynamic effects on output and unemployment.

While the disturbances are *defined* by the identification restrictions, we believe that they can be given a simple economic interpretation. Namely, we interpret the disturbances that have a temporary effect on output as being mostly demand disturbances, and those that have a permanent effect on output as mostly supply disturbances. We present a simple model in which this interpretation is warranted and use it to discuss the justification for, as well as the limitations of, this interpretation.

Under these identification restrictions and this economic interpretation, we obtain the following characterization of fluctuations: demand disturbances have a hump-shaped effect on both output and unemployment; the effect peaks after a year and vanishes after two to three years. Up to a scale factor, the dynamic effect on unemployment of demand disturbances is a mirror image of that on output. The effect of supply disturbances on output increases steadily over time, to reach a peak after two years and a plateau after five years. "Favorable" supply disturbances may initially increase unemployment. This is followed by a decline in unemployment, with a slow return over time to its original value.

While this dynamic characterization is fairly sharp, the data are not as specific as to the relative contributions of demand and supply disturbances to output fluctuations. On the one hand, we find that the time-series of demand-determined output fluctuations, that is the time-series of output constructed by putting all supply disturbance realizations equal to zero, has peaks and troughs which coincide with most of the NBER troughs and peaks. But, when we turn to variance decompositions of output at various horizons, we find that the respective contributions of supply and demand disturbances are not precisely estimated. For instance, at a forecast horizon of four quarters, we find that, under alternative assumptions, the contribution of demand disturbances ranges from 40 percent to over 95 percent.

The rest of the paper is organized as follows. Section I analyzes identification, and Section II discusses our economic interpretation of the disturbances. Section III discusses estimation, and Section IV characterizes the dynamic effects of demand and supply disturbances on output and unemployment. Section V characterizes the relative contributions of demand and supply disturbances to fluctuations in output and unemployment.

I. Identification

In this section, we show how our assumptions characterize the process followed by output and unemployment, and how this process can be recovered from the data.

We make the following assumptions. There are two types of disturbances affecting unemployment and output. The first has no long-run effect on either unemployment or output. The second has no long-run effect on unemployment, but may have a long-run effect on output. Finally, these two disturbances are uncorrelated at all leads and lags. These restrictions in effect define the two disturbances. As indicated in the introduction, and discussed at length in the next section, we will refer to the first as demand disturbances, and to the second as supply disturbances. How we name the disturbances however is irrelevant for the argument of this section.

The demand and supply components described above are permitted to be serially correlated. Under regularity conditions, each of these components can always be uniquely represented as an invertible distributed lag of serially uncorrelated disturbances. Thus, we can refer to the associated serially uncorrelated disturbances as the demand and supply disturbances themselves: this is without ambiguity or loss of generality. We will then also require a further technical condition: the innovations in the bivariate Wold decomposition of output growth and unemployment are linear combinations of these underlying demand and supply disturbances.

We now derive the joint process followed by output and unemployment implied by our assumptions. Let Y and U denote the logarithm of GNP and the level of the unemployment rate, respectively, and let e_d and e_s be the two disturbances. Let X be the vector $(\Delta Y, U)'$ and e be the vector of disturbances $(e_d, e_s)'$. The assumptions above imply that X follows a stationary process given by:

$$(1)\quad X(t) = A(0)e(t) + A(1)e(t-1) + \cdots$$

$$= \sum_{j=0}^{\infty} A(j)e(t-j),$$

$$\mathrm{Var}(e) = I,$$

where the sequence of matrices A is such that its upper left-hand entry, $a_{11}(j)$, $j = 1,2,\ldots$, sums to zero.

Equation (1) gives Y and U as distributed lags of the two disturbances, e_d and e_s. Since these two disturbances are assumed to be uncorrelated, their variance covariance matrix is diagonal; the assumption that the covariance matrix is the identity is then simply a convenient normalization. The contemporaneous effect of e on X is given by $A(0)$; subsequent lag effects are given by $A(j)$, $j \geq 1$. As X has been assumed to be stationary, neither disturbance has a long-run effect on either unemployment, U, or the rate of change in output, ΔY. The restriction $\sum_{j=0}^{\infty} a_{11}(j) = 0$ implies that e_d also has no effect on the *level* of Y itself. To see why this is, notice that $a_{11}(j)$ is the effect of e_d on ΔY after j periods, and therefore, $\sum_{j=0}^{k} a_{11}(j)$ is the effect of e_d on Y itself after k periods. For e_d to have no effect on Y in the long run, we must have then that $\sum_{j=0}^{\infty} a_{11}(j) = 0$.

We now show how to recover this representation from the data. Since X is stationary, it has a Wold-moving average representation:

$$(2)\quad X(t) = \nu(t) + C(1)\nu(t-1) + \cdots$$

$$= \sum_{j=0}^{\infty} C(j)\nu(t-j),$$

$$\mathrm{Var}(\nu) = \Omega.$$

This moving average representation is unique and can be obtained by first estimating and then inverting the vector autoregressive representation of X in the usual way.

Comparing equations (1) and (2) we see that ν, the vector of innovations, and e, the vector of original disturbances, are related by $\nu = A(0)e$, and that $A(j) = C(j)A(0)$, for all j. Thus knowledge of $A(0)$ allows one to recover e from ν, and similarly to obtain $A(j)$ from $C(j)$.

Is $A(0)$ identified? An informal argument suggests that it is. Equations (1) and (2) imply that $A(0)$ satisfies: $A(0)A(0)' = \Omega$, and that the upper left-hand entry in $\sum_{j=0}^{\infty} A(j) = (\sum_{j=0}^{\infty} C(j))\mathrm{A}(0)$ is 0. Given Ω, the first relation imposes three restrictions on the four elements of $A(0)$; given $\sum_{j=0}^{\infty} C(j)$, the other implication imposes a fourth restriction. This informal argument is indeed correct. A rigorous and constructive proof, which we actually use to obtain $A(0)$ is as follows: Let S denote the unique lower triangular Choleski factor of Ω. Any matrix $A(0)$ such that $A(0)A(0)' = \Omega$ is an orthonormal transformation of S. The restriction that the upper left-hand entry in $(\sum_{j=0}^{\infty} C(j))A(0)$ be equal to 0 is an orthogonality restriction that then uniquely determines this orthonormal transformation.[2]

[2] Notice that identification is achieved by a long-run restriction. This raises a knotty technical issue. Without precise prior knowledge of lag lengths, inference and restrictions on the kind of long-run behavior we are interested in here is delicate. See for instance Christopher Sims (1972); we are extrapolating here from Sims's results which assume strictly exogenous regressors. Similar problems may arise in the VAR case, although the results of Kenneth Berk (1974) suggest otherwise. Nevertheless, we can generalize our long-run restriction to one that applies to some neighborhood of

In summary, our procedure is as follows. We first estimate a vector autoregressive representation for X, and invert it to obtain (2). We then construct the matrix $A(0)$; and use this to obtain $A(j)=C(j)A(0)$, $j=0,1,2,\ldots$, and $e_t=A(0)^{-1}\nu_t$. This gives output and unemployment as functions of current and past demand and supply disturbances.

II. Interpretation

Interpreting residuals in small dimensional systems as "structural" disturbances is always perilous, and our interpretation of disturbances as supply and demand disturbances is no exception. We discuss various issues in turn.

Our interpretation of disturbances with permanent effects as supply disturbances, and of disturbances with transitory effects as demand disturbances is motivated by a traditional Keynesian view of fluctuations. For illustrative purposes, as well as to focus the discussion below, we now provide a simple model which delivers those implications. The model is a variant of that in Stanley Fischer (1977):

$$(3)\qquad Y(t)=M(t)-P(t)+a\cdot\theta(t),$$

$$(4)\qquad Y(t)=N(t)+\theta(t),$$

$$(5)\qquad P(t)=W(t)-\theta(t),$$

$$(6)\qquad W(t)=W|\{E_{t-1}N(t)=\overline{N}\}.$$

The variables Y, N, and θ denote the log of output, employment, and productivity, respectively. Full employment is represented by $\overline{N}$; and P, W, and M are the log of the price level, the nominal wage, and the money supply.

Equation (3) states that aggregate demand is a function of real balances and productivity. Notice that productivity is allowed to affect aggregate demand directly; it can do so through investment demand for example, in which case $a>0$. Equation (4) is the production function: it relates output, employment, and productivity, and assumes a constant returns-to-scale technology. Equation (5) describes price-setting behavior, and gives the price level as a function of the nominal wage and of productivity. Finally the last equation, (6), characterizes wage-setting behavior in the economy: the wage is chosen one period in advance, and is set so as to achieve (expected) full employment.

To close the model, we need to specify how M and θ evolve. We assume that they follow:

$$(7)\qquad M(t)=M(t-1)+e_d(t),$$

$$(8)\qquad \theta(t)=\theta(t-1)+e_s(t),$$

where e_d and e_s are the serially uncorrelated and pairwise orthogonal demand and supply disturbances. Define unemployment U to be $\overline{N}-N$; solving for unemployment and output growth then gives:

$$\Delta Y=e_d(t)-e_d(t-1)+a\cdot(e_s(t)-e_s(t-1))+e_s(t),$$

$$U=-e_d(t)-a\cdot e_s(t).$$

These two equations clearly satisfy the restrictions in equation (1) of the previous section. Due to nominal rigidities, demand disturbances have short-run effects on output and unemployment, but these effects disappear over time. In the long run, only supply, that is, productivity disturbances here, affect output. Neither of the disturbances have a long-run impact on unemployment.

This model is clearly only illustrative. More complex wage and price dynamics, such as in John Taylor (1980), will also satisfy the long-run properties embodied in equation (1). This model is nevertheless a useful vehicle to discuss the limitations of our interpretation of permanent and transitory disturbances.

frequency zero, instead of just a restriction at the point zero. Under appropriate regularity conditions, we can show that our results are the limit of those from that kind of restriction, as the neighborhood shrinks to zero.

Granting our interpretation of these disturbances as demand and supply disturbances, one may nevertheless question the assumption that the two disturbances are uncorrelated at all leads and lags. We think of this as a nonissue. The model makes clear that this orthogonality assumption does not eliminate for example the possibility that supply disturbances directly affect aggregate demand. Put another way, the assumption that the two disturbances are uncorrelated does not restrict the channels through which demand and supply disturbances affect output and unemployment.

Again granting our interpretation of these disturbances as demand and supply disturbances, one may argue that even demand disturbances have a long-run impact on output: changes in the subjective discount rate, or changes in fiscal policy may well affect the savings rate, and subsequently the long-run capital stock and output. The presence of increasing returns, and of learning by doing, also raise the possibility that demand disturbances may have some long-run effects. Even if not, their effects through capital accumulation may be sufficiently long lasting as to be indistinguishable from truly permanent effects in a finite data sample. We agree that demand disturbances may well have such long-run effects on output. However, we also believe that if so, those long-run effects are small compared to those of supply disturbances. To the extent that this is true then, our decomposition is "nearly correct" in the following sense: in a sequence of economies where the size of the long-run effect of demand disturbances becomes arbitrarily small relative to that of supply, the correct identifying scheme approaches that which we actually use. This result is proven in the technical appendix.

This raises a final set of issues, one inherent in the estimation and interpretation of any low-dimensional dynamic system. It is likely that there are in fact many sources of disturbances, each with different dynamic effects on output and on unemployment, rather than only two as we assume here. Certainly if there are many supply disturbances, some with permanent and others with transitory effects on output, together with many demand disturbances, some with permanent and others with transitory effects, and if they all play an equally important role in aggregate fluctuations, our decomposition is likely to be meaningless. A more interesting case is that where all the supply disturbances have permanent output effects, and where all the demand disturbances have only transitory output effects. One may then hope that, in this case, what we present as "the" demand shock represents an average of the dynamic effects of the different shocks (in the sense of Clive Granger and M. J. Morris, 1976, for example), and similarly for supply shocks. This however is not true in general: a simple counterexample that illustrates this is provided in the technical appendix. However, we also present in the appendix necessary and sufficient conditions such that an aggregation proposition does hold. Those conditions will be satisfied if for instance, the economy is subject to only one supply disturbance but many demand disturbances, where each of the demand disturbances has different dynamic effects on output, but all the demand disturbances leave unaffected the dynamic relation between output and unemployment. That demand disturbances should leave the relation between output and unemployment nearly unaffected is highly plausible. That the economy is subject to only one, or at least to one dominant, source of supply disturbances is more questionable. If there are many supply disturbances of roughly equal importance, and if, as is likely, each of them affects the dynamic relation between unemployment and output, our decomposition is likely to be meaningless.

In summary, our interpretation of the disturbances is subject to various caveats. Nevertheless we believe that interpretation to be reasonable and useful in understanding the results below. We now briefly discuss the relation of our paper to others on the same topic. We first examine how our approach relates to the business-cycle-versus-trend distinction.

Following estimation, we can construct two output series, a series reflecting only the effects of supply disturbances, obtained by setting all realizations of the demand disturbances to zero, and a series reflecting only the effects of demand disturbances, obtained by setting supply realizations to zero. By construction, the first series, the supply component of output, will be nonstationary while the second, the demand component, is stationary.[3]

A standard distinction in describing output movements is the "business cycle versus trend" distinction. While there is no standard definition of these components, the trend is usually taken to be that part of output that would realize, were all prices perfectly flexible; business cycles are then taken to be the dynamics of actual output around its trend.[4]

It is tempting to associate the first series we construct with the "trend" component of output and the second series with the "business cycle" component. In our view, that association is unwarranted. If prices are in fact imperfectly flexible, deviations from trend will arise not only from demand disturbances, but also from supply disturbances: business cycles will occur due to both supply and demand disturbances. Put another way, supply disturbances will affect both the business cycle and the trend component. Identifying separately business cycles and trend is likely to be difficult, as the two will be correlated through their joint dependence on current and past supply disturbances.

With this discussion in mind, we now review the approaches to identification used by others.

Campbell and Mankiw (1987b) assume the existence of two types of disturbances, "trend" and "cycle" disturbances, which are assumed to be uncorrelated. Their identifying restriction is then that trend disturbances do not affect unemployment. The discussion above suggests that this assumption of zero correlation between cycle and trend components is unattractive; if their two disturbances are instead reinterpreted as supply and demand disturbances, respectively, the identifying restriction that supply disturbances do not affect unemployment is equally unattractive.

Clark (1988) also assumes the existence of "trend" and "cycle" disturbances, and also assumes that "trend" disturbances do not affect unemployment but allows for contemporaneous correlation between trend and cycle disturbances. While this may be seen as an improvement over Campbell and Mankiw, it still severely constrains the dynamic effects of disturbances on output and unemployment in ways that are difficult to interpret.

The paper closest to ours is that of Evans (1987). Evans assumes two disturbances, "unemployment" and "output" disturbances, which can be reinterpreted as supply and demand disturbances, respectively. By assuming the existence of a reduced form identical to equation (2) above, he also assumes that neither supply nor demand disturbances have a long-run effect on unemployment, but that both may have a long-run effect on the level of output. However, instead of using the long-run restriction that we use here, he assumes that supply disturbances have no contemporaneous effect on output. We find this restriction less appealing as a way of achieving identification; it should be clear however that our paper builds on Evans' work.

III. Estimation

We need to confront one final problem before estimation. The representation we use in Section I assumes that both the level of unemployment and the first difference of the logarithm of GNP are stationary around given levels. Postwar-U.S. data however suggest instead both a small but steady increase in the average unemployment rate over the sample, as well as a decline in the average

[3] There is a technical subtlety here: strictly speaking, the fact that the sum of coefficients approaches zero is a necessary but not sufficient condition for the demand component to be stationary. However it turns out to be sufficient when unemployment and output growth are individually ARMA processes. This is proven in Quah (1988).

[4] A precise definition would obviously be tricky but is not needed for our argument. In models with imperfect information, this would be the path of output, absent imperfect information. In models with nominal rigidities, this would be the path of output, absent nominal rigidities. In models that assume market clearing and perfect information, such as in Edward Prescott (1987), the distinction between business cycles and trend is not a useful one.

growth rate of GNP since the mid-1970s.[5] This raises two issues.

The first is that our basic assumptions may be wrong in fundamental ways. For instance, unemployment might in fact be nonstationary, and affected even in the long run by demand and supply disturbances. This is predicted by models with a "hysteresis" effect, as developed in Blanchard and Lawrence Summers (1986), and used by them to explain European unemployment. This property also obtains in some recent growth models with increasing returns to scale, where changes in the savings rate may affect not only the level but also the growth rate of output. While we cannot claim that such effects are not present here, we are willing to assume that their importance is minimal, for the period and the economy at hand.

Next, there is the issue of how to handle the apparent time trend in unemployment, and the apparent slowdown in growth since the mid-1970s. There is no clean solution for this, and we take an eclectic approach.[6] To focus the discussion, we present as a base case the results from estimation allowing for a change in the growth rate of output, and for a secular increase in the unemployment rate, as captured by a fitted-linear time-trend regression line. There are three other cases of interest: (a) there is no change in the growth rate of output, but there is a secular change in the unemployment rate; (b) there is no secular trend in the unemployment rate, but there is a break in the average growth rate of output; and finally, (c) there is neither a change in the growth rate of output nor a secular change in the unemployment rate.

A VAR system in real GNP growth (ΔY) and the unemployment rate (U), allowing for eight lags is estimated using observations from 1950:2 through 1987:4.[7] The GNP data are quarterly; the monthly unemployment data are averaged to provide quarterly observations. Evans (1987) has estimated essentially the same bivariate VAR representation, although he uses instead the aggregate civilian unemployment rate. He has also tested the stationarity assumptions that we use here. The properties of the VAR representation and of the moving average representation found by direct inversion do not have any meaning within our framework, so we do not discuss those further here.

The mean growth rates for output are 3.62 percent and 2.43 percent, at an annual rate, over 1948:2 through 1973:4, and 1974:1 through 1987:4, respectively. This break point is chosen to coincide with the first OPEC oil shock. The fitted-time-trend regression coefficient for the unemployment rate series is 0.019, which implies a secular increase of 2.97 percentage points over the sample period. When we allow for a change in the output growth, we simply remove the different sample means before estimating the vector autoregression; similarly when we allow for a secular change in the unemployment rate, the fitted-trend line is removed before VAR analysis.

It turns out that the results for cases (a)–(c) are qualitatively similar to those for the base case. More precisely, the moving average responses to demand and supply disturbances are sufficiently close to those of the base case in their main features; the principal differences lie in the magnitudes of the responses. These differences are notable only in forecast error variance decompositions; we will therefore present four such decomposition tables for the different cases below. Because of the similarity in the other qualitative features however, and to conserve space, we will present results for the impulse

[5]The increase in the unemployment rate, sometimes attributed to demographic changes, is evident even in the relatively homogenous labor group on which we focus our attention. We use the seasonally adjusted unemployment rate for Males, age 20 and over. This is from the U.S. Department of Labor, Bureau of Labor Statistics (BLS), 1982, and BLS Table A-39, February issues.

[6]See for example Pierre Perron (1987) and Lawrence Christiano (1988) on the statistical evidence for and against a break in average growth over the postwar period.

[7]Estimation with twelve lags produced little difference in the results. We also experimented with omitting the first five years, as the Korean War experience seemed anomalous. Again, the empirical results remain practically unchanged.

responses and historical decompositions only for the base case.[8]

We turn next to the dynamic effects of demand and supply disturbances.

IV. Dynamic Effects of Demand and Supply Disturbances

The dynamic effects of demand and supply disturbances are reported in Figures 1 and 2. The vertical axes in Figures 1 and 2 denote simultaneously the log of output and the rate of unemployment; the horizontal axis denotes time in quarters. Figures 3–6 provide the same information, but now with one standard deviation bands around the point estimates.[9]

Demand disturbances have a hump-shaped effect on output and unemployment. Their effects peak after two to four quarters. The effects of demand then decline to vanish after about three to five years. The responses in output and unemployment are mirror images of each other; we return to this aspect of the results below after discussing the effects of supply disturbances.

The output response is smallest when the raw data are used, without allowing for a break or a secular change in unemployment (case c, not shown); it also decays the most rapidly in this case. Once a change in the average growth rate of output is allowed, the treatment of possible secular changes in unemployment seems to be relatively unimportant for the responses to demand disturbances.

These dynamic effects are consistent with a traditional view of the dynamic effects of aggregate demand on output and unemployment, in which movements in aggregate demand build up until the adjustment of prices and wages leads the economy back to equilibrium.

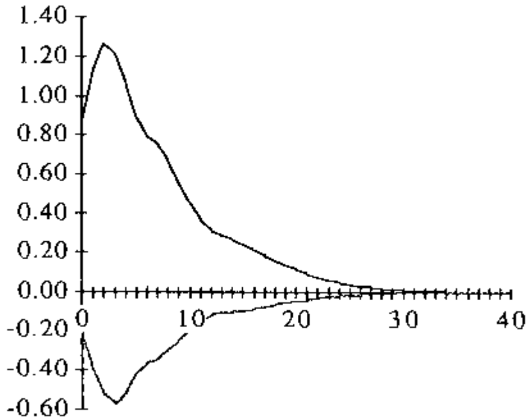

FIGURE 1. RESPONSE TO DEMAND, — = OUTPUT, --- = UNEMPLOYMENT

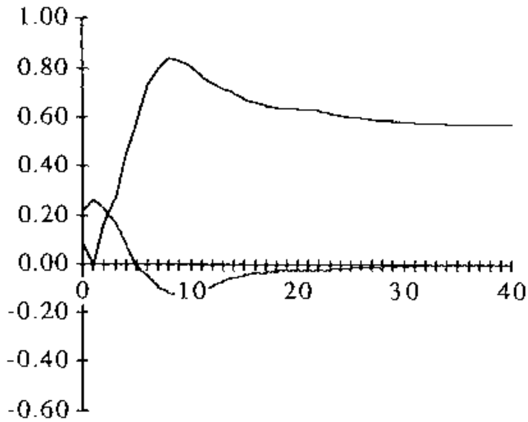

FIGURE 2. RESPONSE TO SUPPLY, — = OUTPUT, --- = UNEMPLOYMENT

Supply disturbances have an effect on the level of output which cumulates steadily over time. In the base case, the peak response is about eight times the initial effect and takes place after eight quarters. The effect decreases to stabilize eventually. For good statistical reasons, the long-run impact is imprecisely estimated. The dynamic response in unemployment is quite different: a positive supply disturbance (that is, a supply disturbance that has a positive long-run effect on output) initially increases unemploy-

[8]The other graphs are available from the authors upon request.

[9]More precisely, these boundaries are separated from the point estimate by the square root of mean squared deviations in each direction, over 1000 bootstrap replications. Thus the bands need not be and indeed are not symmetric. By construction, they will of course necessarily include the point estimate. In each case, pseudohistories are created by drawing with replacement from the empirical distribution of the VAR innovations.

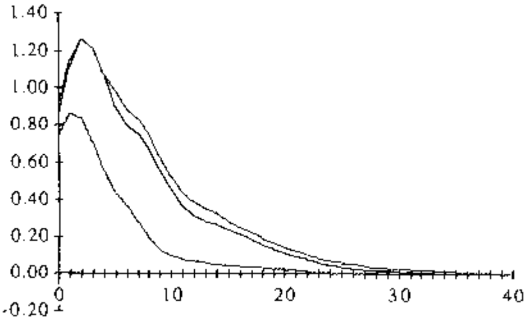

FIGURE 3. OUTPUT RESPONSE TO DEMAND

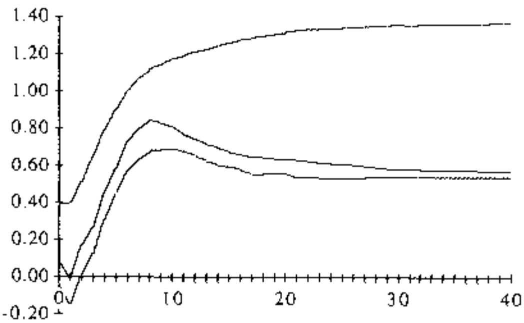

FIGURE 4. OUTPUT RESPONSE TO SUPPLY

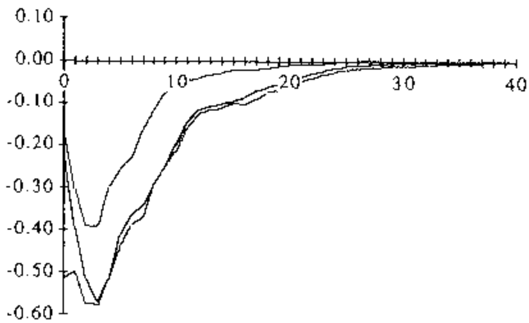

FIGURE 5. UNEMPLOYMENT RESPONSE TO DEMAND

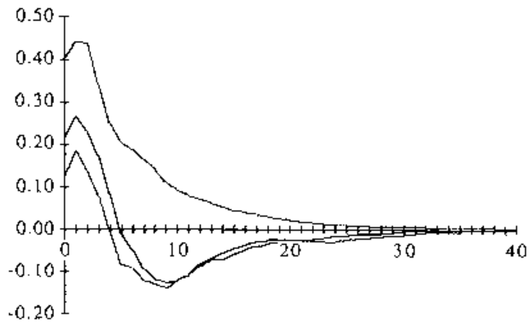

FIGURE 6. UNEMPLOYMENT RESPONSE TO SUPPLY

ment slightly. Following this increase, the effect is reversed after a few quarters, and unemployment slowly returns to its original steady-state value. The dynamic effects of a supply disturbance on unemployment are largely over by about five years.

The qualitative results are similar across all alternative treatments of breaks and time trends. The only significant difference appears in the initial unemployment response to demand disturbances: in the case when neither break nor time trend is permitted, the response is initially negative rather than positive as in the base case. The one standard deviation band does however include positive values.

The response of unemployment and output are suggestive of the presence of rigidities, both nominal and real. Nominal rigidities can explain why in response to a positive supply shock, say an increase in productivity, aggregate demand does not initially increase enough to match the increase in output needed to maintain constant unemployment; real wage rigidities can explain why increases in productivity can lead to a decline in unemployment after a few quarters which persists until real wages have caught up with the new higher level of productivity.

Figures 1 and 2 also shed interesting light on the relation between changes in unemployment and output known as Okun's law. The textbook value of Okun's coefficient is about 2.5. Under our interpretation, this coefficient is a mongrel coefficient, as the joint behavior of output and unemployment depends on the type of disturbance affecting the economy. In the case of demand disturbances, Figure 1 suggests that there is indeed a tight relation between output and unemployment. At the peak responses, the graph suggests an implied coefficient between output and unemployment that is slightly greater than 2. In the case of supply disturbances, there is no such close relation between output and unemployment. In the short run, output increases, unemployment may rise or fall; in the long run, output remains higher whereas—by assumption—unemployment returns to its initial value. In the intervening period, unemployment and output deviations are of opposite sign. At the peak responses, Figure 2 suggests an implied coefficient slightly exceeding four, higher in absolute value than Okun's coefficient. That the absolute value of the coefficient is higher for supply disturbances than for demand disturbances is exactly what we expect. Supply disturbances are likely to affect the relation between output and employment, and to increase output with little or no change in employment.

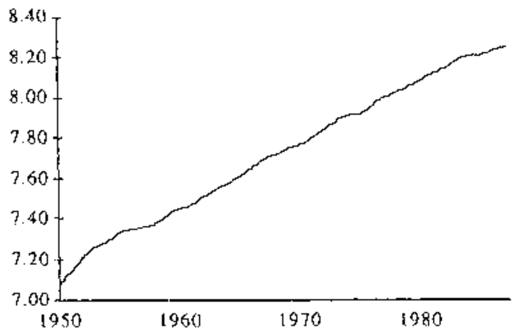

FIGURE 7. OUTPUT FLUCTUATIONS ABSENT DEMAND

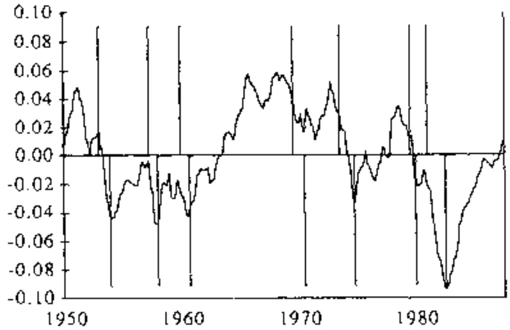

FIGURE 8. OUTPUT FLUCTUATIONS DUE TO DEMAND

V. Relative Contributions of Demand and Supply Disturbances.

Having shown the dynamic effects of each type of disturbance, the next step is to assess their relative contribution to fluctuations in output and unemployment. We do this in two ways. The first is informal, and entails a comparison of the historical time-series of the demand component of output to the NBER chronology of business cycles. The second examines variance decompositions of output and unemployment in demand and supply disturbances at various horizons.

A. *Demand Disturbances and NBER Business Cycles*

From estimation of the joint process for output and unemployment, and our identifying restrictions, we can form the "demand components" of output and unemployment. These are the time paths of output and unemployment that would have obtained in the absence of supply disturbances. Similarly, by setting demand innovations to zero, we can generate the time-series of "supply components" in output and unemployment. From the identifying restriction that demand disturbances have no long-run effect on output, the resulting series of the demand component in the level of output is stationary. By the same token, both the demand and supply components of unemployment are stationary.

The time-series for these components are presented in Figures 7 through 10. Superimposed on these time series are the NBER peaks and troughs. Peaks are drawn as vertical lines above the horizontal axis, troughs as vertical lines below the axis.

The peaks and troughs of the demand component in output match closely the NBER peaks and troughs. The two recessions of 1974–1975 and 1979–1980 deserve special mention. Our decomposition attributes them in about equal proportions to adverse supply and demand disturbances. This is best shown by giving the estimated values of the supply and demand innovations over these periods. These are collected in Table 1. The recession of 1974–75 is therefore explained by an initial string of negative supply disturbances, and then of

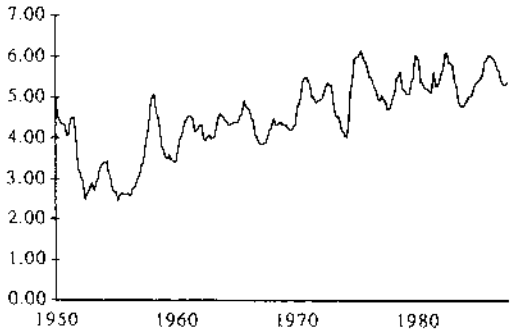

FIGURE 9. UNEMPLOYMENT FLUCTUATIONS ABSENT DEMAND

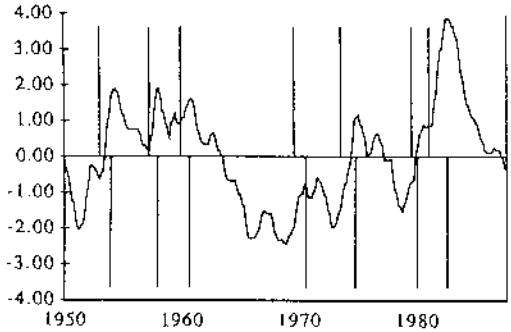

FIGURE 10. UNEMPLOYMENT FLUCTUATIONS DUE TO DEMAND

negative demand disturbances. Similarly, the 1979–80 recession is first dominated by a large negative supply disturbance in the second quarter of 1979, and then a large negative demand disturbance a year later. Without appearing to interpret every single residual, we find these estimated sequences of demand and supply disturbances consistent with less formal descriptions of these episodes.[10]

TABLE 1—DEMAND AND SUPPLY INNOVATIONS[a] BASE CASE[b]

Quarter	Demand (Percent)	Supply (Percent)
1973–3	−0.8	−1.9
1973–4	0.3	−0.4
1974–1	−0.7	−0.0
1974–2	0.5	−1.5
1974–3	−1.8	−1.0
1974–4	−0.7	1.1
1975–1	−1.5	2.8
–		
1979–1	−0.5	−0.3
1979–2	−0.4	−1.7
1979–3	0.7	0.6
1979–4	−0.8	0.2
1980–1	0.2	2.0
1980–2	−3.2	1.9

Notes:
[a]The identified innovations are obtained by applying the transformation of Section I to the fitted-VAR residuals. By construction, the standard deviations of these innovations are equal to 1 percent.
[b]The estimated innovations for the other cases follow the same pattern as above.

Notice that the supply component in output, presented in Figure 7, is clearly not a deterministic trend. It exhibits slower growth in the late 1950s, as well as in the 1970s.

Figures 9 and 10 give the supply and demand components in *unemployment*. Unemployment fluctuations due to demand correspond closely to those in the demand component of GNP. This is consistent with our earlier finding on the mirror image moving average responses of unemployment and output growth to demand disturbances. The model attributes substantial fluctuation in unemployment to supply disturbances, again with increases in the late 1950s and around the time of the oil disturbances of the 1970s.[11]

[10]Formal evidence of a slightly different nature is also available. In Blanchard and Watson (1986), evidence from four time-series is used to decompose fluctuations into supply and demand disturbances. There the recession of 1975 is attributed in roughly equal proportions to adverse demand and supply disturbances, that of 1980 mostly to demand disturbances. To see how much our characterization of the dynamic effects of demand and supply disturbances depend on the 1973–76 episode, we reestimated the model, leaving out 1973–1 to 1976–4. The estimated dynamic effects of both demand and supply disturbances were nearly identical to those described above.

[11]By construction, the supply component of unemployment is close to actual unemployment for the first few observations in the sample. Thus, the large decrease from 1950 to 1952 in the supply component simply reflects the actual movement in unemployment in this period. In light of this, we reestimated the model from 1955–2 through the end of our sample. We found little change in the empirical results.

TABLE 2—VARIANCE DECOMPOSITION OF OUTPUT AND UNEMPLOYMENT (CHANGE IN OUTPUT GROWTH AT 1973/1974; UNEMPLOYMENT DETRENDED)

Percentage of Variance Due to Demand:		
Horizon (Quarters)	Output	Unemployment
1	99.0	51.9
	(76.9, 99.7)	(35.8, 77.6)
2	99.6	63.9
	(78.4, 99.9)	(41.8, 80.3)
3	99.0	73.8
	(76.0, 99.6)	(46.2, 85.6)
4	97.9	80.2
	(71.0, 98.9)	(49.7, 89.5)
8	81.7	87.3
	(46.3, 87.0)	(53.6, 92.9)
12	67.6	86.2
	(30.9, 73.9)	(52.9, 92.1)
40	39.3	85.6
	(7.5, 39.3)	(52.6, 91.6)

TABLE 2A—VARIANCE DECOMPOSITION OF OUTPUT AND UNEMPLOYMENT (NO DUMMY BREAK, TIME TREND IN UNEMPLOYMENT)

Percentage of Variance Due to Demand:		
Horizon (Quarters)	Output	Unemployment
1	83.8	79.7
	(59.4, 93.9)	(55.3, 92.0)
2	87.5	88.2
	(62.8, 95.4)	(58.9, 95.2)
3	83.4	93.5
	(58.8, 93.3)	(61.3, 97.5)
4	78.9	95.7
	(53.5, 90.0)	(63.9, 98.2)
8	52.5	88.9
	(31.4, 68.6)	(63.5, 94.5)
12	37.8	79.7
	(21.3, 51.4)	(58.8, 90.3)
40	18.7	75.9
	(7.4, 23.5)	(56.9, 88.6)

B. *Variance Decompositions*

While the above empirical evidence is suggestive, a more formal statistical assessment can be given by computing variance decompositions for output and unemployment at various horizons.

Tables 2, and 2A–C give this variance decomposition for the different cases. The table has the following interpretation. Define the k quarter-ahead forecast error in output as the difference between the actual value of output and its forecast from equation (2) as of k quarters earlier. This forecast error is due to both unanticipated demand and supply disturbances in the last k quarters. The number for output at horizon k, $k = 1, \ldots, 40$ gives the percentage of variance of the k-quarter ahead forecast error due to demand. The contribution of supply, not reported, is given by 100 minus that number. A similar interpretation holds for the numbers for unemployment. The numbers in parentheses are one standard deviation bands, surrounding the point estimate.[12]

[12]Again, these bands are asymmetric, and obtained as described above.

TABLE 2B—VARIANCE DECOMPOSITION OF OUTPUT AND UNEMPLOYMENT (CHANGE IN OUTPUT GROWTH AT 1973/1974; NO TREND IN UNEMPLOYMENT)

Percentage of Variance Due to Demand:

Horizon (Quarters)	Output	Unemployment
1	99.3	50.7
	(75.0, 99.8)	(32.0, 79.9)
2	99.7	63.2
	(77.6, 99.9)	(36.6, 83.3)
3	99.4	73.4
	(76.1, 99.7)	(40.8, 88.3)
4	98.6	80.0
	(72.9, 99.2)	(44.3, 91.1)
8	86.3	88.4
	(53.2, 91.5)	(50.0, 94.6)
12	75.5	88.9
	(40.9, 83.0)	(49.9, 94.6)
40	50.4	90.0
	(12.5, 54.8)	(49.7, 95.0)

TABLE 2C—VARIANCE DECOMPOSITION OF OUTPUT AND UNEMPLOYMENT (NO DUMMY BREAK, NO TREND IN UNEMPLOYMENT)

Percentage of Variance Due to Demand:

Horizon (Quarters)	Output	Unemployment
1	45.2	99.8
	(20.1, 77.6)	(76.6, 100.0)
2	50.2	98.3
	(23.4, 79.9)	(72.8, 99.3)
3	44.2	92.7
	(20.4, 77.0)	(67.1, 97.8)
4	38.9	85.9
	(17.1, 72.7)	(62.7, 95.8)
8	19.6	60.5
	(8.8, 54.4)	(44.3, 89.6)
12	12.9	47.6
	(6.5, 43.5)	(35.2, 87.8)
40	5.2	40.5
	(2.4, 17.7)	(31.4, 87.1)

Our identifying restrictions impose only one restriction on the variance decompositions, namely that the contribution of supply disturbances to the variance of output tends to unity as the horizon increases. All other aspects are unconstrained.

Two principal conclusions emerge from these tables.

First, the data do not give a precise answer as to the relative contribution of demand and supply disturbances to movements in *output* at short and medium-term horizons. The results vary across alternative treatments of break and trend. In the base case, the relative contribution of demand disturbances to output fluctuations, at a four quarters horizon, in 98 percent. This contribution falls to 79 percent when no break is allowed but there is a time trend in unemployment, remains about the same when a break is allowed in output growth but there is no trend in the unemployment rate. When neither a break nor a trend is permitted, it is only 39 percent. Next, the standard error bands are quite large in each case, ranging from 71 to 99 percent in the base case, 54 to 90 percent in case A, 73 to 99 percent in case B, and 17 to 73 percent in case C. Evidently when a break is permitted in output growth, the treatment of the trend in unemployment appears to be quite unimportant. These cases are also when the demand contribution is more precisely estimated. Despite the differences across estimates, and the uncertainty associated with each set, we view the results as suggesting an important role for demand disturbances in the short run.

Second, estimates of the relative contribution of the different disturbances to *unemployment* do not appear to vary a great deal across alternative treatments of break and trend. The contribution of demand disturbances, four quarters ahead, to unemployment fluctuations varies from 80 to 96 percent. In the base case, the one standard error band ranges from 50 to 90 percent with a point estimate of 80 percent. In all cases, the demand disturbance appears to be quite important for unemployment fluctuations at all horizons.

VI. Conclusion and Extensions

We have assumed the existence of two types of disturbances generating unemployment and output dynamics, the first type having permanent effects on output, the second having only transitory effects. We have argued that these two types of disturbances could usefully be interpreted as supply and demand shocks. Under that interpretation, we have concluded that demand disturbances have a hump-shaped effect on output and unemployment which disappears after approximately two to three years, and that supply disturbances have an effect on output which cumulates over time to reach a plateau after five years. We have also concluded that demand disturbances make a substantial contribution to output fluctuations at short- and medium-term horizons; however, the data do not allow us to quantify this contribution with great precision.

While we find this simple exercise to have been worthwhile, we also believe that further work is needed, especially to validate and refine our identification of shocks as supply and demand shocks. We have in mind two specific extensions. The first is to examine the co-movements of what we have labeled the demand and supply components of GNP with a larger set of macroeconomic variables. Preliminary results appear to confirm our interpretation of shocks. We find in particular the supply component of GNP to be positively correlated with real wages at high to medium frequencies, while no such correlation emerges for the demand component.[13] The second extension is to enlarge the system to one in four variables, unemployment, output, prices, and wages. This would also allow examination of different questions from an alternative perspective, as in Blanchard (1989). As one might expect, wage and price data will help identify more explicitly supply and demand disturbances. Research by Jordi Gali (1988), Sung-in Jun (1988), and Matthew Shapiro and Watson (1988) has already extended our work in that particular direction.

Technical Appendix

This technical appendix discusses further and establishes the claims made in the section on interpretation.

First, we asserted in the text that our identification scheme is approximately correct even when both disturbances have permanent effects on the level of output, provided that the long-run effect of demand on output is small. We now prove this.

The first element of the model, output growth, has the moving average representation in demand and supply disturbances:

$$\Delta Y_t = a_{11}(L)e_{dt} + a_{12}(L)e_{st},$$

where $a_{11}(1)$ is the cumulative effect on the level of output Y of the disturbance e_d. The moving average representation $C(L)$, together with the innovation covariance matrix Ω, is related to our desired interpretable representation through some identifying ma-

[13] The methodology and results will be described in a future paper. The statement in the text refers to the sum of correlations from lags -5 to $+5$ between the supply innovation derived in this paper and the innovations in real wages obtained from univariate ARIMA estimation.

trix S, such that:

$$SS' = \Omega, \quad \text{and} \quad A(L) = C(L)S.$$

The model is identified by choosing a unique identifying matrix S. In the paper, we selected the unique matrix S such that $a_{11}(1) = 0$.

Let the long-run effect of the demand disturbance be δ instead, where $\delta > 0$ without loss of generality. For each δ, this implies a different identifying matrix $S(\delta)$. Let $|S(\delta) - S(0)| = \max_{j,k}(S_{jk}(\delta) - S_{jk}(0))^2$; this measures the deviation in the implied identifying matrix from that which we use. Since the approximation is thus seen to be a finite-dimensional problem, any matrix norm will induce the same topology, which is all that is needed to study the continuity properties of our identification scheme. All of the empirical results vary continuously in S relative to this topology. Thus, it is sufficient to show that

$$|S(\delta) - S(0)| \to 0 \quad \text{as} \quad \delta \to 0.$$

In words, if an economy has long-run effects in demand that are small but different from zero, our identifying scheme which incorrectly assumes the long-run effects to be zero nevertheless recovers approximately the correct point estimates.

PROOF:

We prove this as follows. Since both $S(0)$ and $S(\delta)$ are matrix square roots of Ω, there exists an orthogonal matrix $V(\delta)$ such that:

$$S(\delta) = S(0)V(\delta), \quad \text{where} \quad V(\delta)V(\delta)' = I.$$

Then the long-run effect of demand is the $(1,1)$ element in the matrix:

$$A(1;\delta) = C(1)S(\delta) = C(1)S(0)V(\delta).$$

But recall that the elements of the first row of $C(1)S(0)$ are respectively, the long-run effects of demand and of supply on the level of output, when the long-run effect of demand is restricted to be zero. Thus for any $V(\delta)$, the new implied long-run effect of demand is simply the long-run effect of supply (under our identifying assumption that the long-run effect of demand is zero) multiplied by the $(2,2)$ element of the orthogonal matrix $V(\delta)$. As δ tends to zero, the $(2,2)$ element of $V(\delta)$ tends continuously to zero as well. But, up to a column sign change, the unique $V(\delta)$ with $(2,2)$ element equal to zero is the identity matrix. This establishes that $S(\delta) \to S(0)$, element by element. Hence, we have shown that $|S(\delta) - S(0)| \to 0$ as $\delta \to 0$. □

Next, we turn to the effects of multiple demand and supply disturbances: Suppose that there is a $p_d \times 1$ vector of demand disturbances f_{dt}, and a $p_s \times 1$ vector of supply disturbances f_{st}, so that:

$$\begin{pmatrix} \Delta Y_t \\ U_t \end{pmatrix} = \begin{pmatrix} B_{11}(L)' & B_{12}(L)' \\ B_{21}(L)' & B_{22}(L)' \end{pmatrix} \begin{pmatrix} f_{dt} \\ f_{st} \end{pmatrix},$$

where B_{jk} are column vectors of analytic functions; B_{j1} has the same dimension as f_d, B_{j2} has the same dimension as f_s, and $B_{11}(z) = (1-z)\beta_{11}(z)$, for some vector of analytic functions β_{11}. Each disturbance has a different distributed lag effect on output and unemployment.

Since our VAR method allows identification of only as many disturbances as observed variables, it is immediate that we will not be able to recover the individual components of $f = (f_d' \, f_s')'$.

To clarify the issues involved, we provide an explicit example where our procedure produces misleading results. Suppose that there is only one supply disturbance and two demand disturbances: $f_{dt} = (f_{d1,t}, f_{d2,t})'$. Suppose further that the first demand disturbance affects only output, while the second demand disturbance affects only unemployment. The supply disturbance affects both output and unemployment. Formally, assume that the true model is:

$$X_t = \begin{pmatrix} \Delta Y_t \\ U_t \end{pmatrix} = \begin{pmatrix} 1-L & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} f_{d1,t} \\ f_{d2,t} \\ f_{st} \end{pmatrix}.$$

An unrestricted VAR representation corresponding to this data generating process is found by applying the calculations in Yu Rozanov (1967), Theorem 10.1, (pp. 44–48). The implied moving average representation is:

$$X_t = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}(2-L) \\ \sqrt{2} & 0 \end{pmatrix} \eta_t, \quad E\eta_t\eta_t' = I.$$

It is straightforward to verify that the matrix covariogram implied by this moving average matches that of the true underlying model. Further, the unique zero of the determinant is 2, and consequently lies outside the unit circle. Therefore this moving average representation is, as asserted, obtained from the vector autoregressive representation of the true model.

However, this moving average does not satisfy our identifying assumption that the "demand" disturbance has only transitory effects on the level of output. We therefore apply our identifying transformation to obtain:

$$X_t = \begin{pmatrix} \frac{1}{2}(1-L) & \frac{1}{2}(3-L) \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e_{dt} \\ e_{st} \end{pmatrix}.$$

This moving average representation is what we would recover if in fact the data are generated by the three disturbances (f_{d1}, f_{d2}, f_s). Notice that while the supply disturbance f_s affects both output growth and unemployment equally and only contemporaneously, we would identify e_s to have a larger effect on output than on unemployment, together with a distributed lag effect on output. Further a positive demand disturbance, restricted to have only a transitory effect on output, is seen to have a contemporaneous negative impact on unemployment. In the true model however, no demand disturbances affect output and unemployment together, either contemporaneously or at any lag. In conclusion, a researcher following our bivariate procedure is likely to be seriously misled when in fact the true underlying model is driven by more than two disturbances. Having seen this, we ask under what circumstances will this mismatch in the number of actual and explicitly modeled disturbances be benign?

We state the necessary and sufficient conditions for this as a theorem which is proved below.

THEOREM: *Let X be a bivariate stochastic sequence generated by*

(i) $X_t = B(L)f_t$;

(ii) $f_t = (f_{dt}'\ f_{st}')'$, *with* $f_d\, p_d \times 1$, $f_s\, p_s \times 1$;

(iii) $Ef_t f_{t-k} = I$ *if* $k = 0$, *and* 0 *otherwise*;

(iv) $B(z) = \begin{pmatrix} B_{11}(z)' & B_{12}(z)' \\ B_{21}(z)' & B_{22}(z)' \end{pmatrix}$;

(v) $B_{11}(z) = (1-z)\beta_{11}(z)$;

(vi) β_{11}, B_{21}, B_{12}, B_{22} *are column vectors of analytic functions*; β_{11} *and* $B_{21}\, p_d \times 1$, B_{12} *and* $B_{22}\, p_s \times 1$;

(vii) BB^* *is full rank on* $|z| = 1$, *where* $*$ *denotes complex conjugation followed by transposition.*

Then there exists a bivariate moving average representation for X, $X_t = A(L)e_t$, such that:

(viii) $A(z) = \begin{pmatrix} a_{11}(z) & a_{12}(z) \\ a_{21}(z) & a_{22}(z) \end{pmatrix}$, *with* a_{11}, a_{12}, a_{21}, a_{22} *scalar functions*, det $A \neq 0$ *for all* $|z| \leq 1$;

(ix) $a_{11}(z) = (1-z)\alpha_{11}(z)$, *with* α_{11} *analytic on* $|z| \leq 1$; *and*

(x) $e_t = \begin{pmatrix} e_{dt} \\ e_{st} \end{pmatrix}$, $Ee_t e_{t-k} = I$ *if* $k = 0$, *and is* 0 *otherwise.*

In the bivariate representation, e_d is orthogonal to f_s, and e_s is orthogonal to f_d, at all leads and lags if and only if there exists a pair of scalar functions γ_1, γ_2 such that:

$$B_{21} = \gamma_1 \cdot \beta_{11},$$

$$B_{22} = \gamma_2 \cdot B_{12}.$$

Conditions (i)–(vii) describe the true data generating process for the observed data in output growth and unemployment. There are p_d demand and p_s supply disturbances; (v) expresses the requirement that demand disturbances have only transitory effects on the *level* of output. Condition (vii) is a regularity condition that allows the existence of a VAR mean square approximation. The moving average recovered by our VAR procedure is described by (viii)–(x): the theorem guarantees that there always exists such a representation.

The second part of the theorem establishes necessary and sufficient conditions on the underlying model such that the bivariate identification procedure does not inappropriately confuse demand and supply disturbances. In words, correct identification is possible if and only if the individual distributed lag responses in output growth and unemployment are sufficiently similar across the different demand disturbances, and across the different supply disturbances. This does *not* mean that the dynamic responses in output growth and unemployment across demand disturbances must be identical or proportional, simply that they differ up to a scalar lag distribution.

Thus even though in general a bivariate procedure is misleading, there are important and reasonable sets of circumstances under which our technique provides the "correct" answers. For instance, suppose that there is only one supply disturbance but multiple demand disturbances. Suppose further that each of the demand components in the level of output has the same distributed lag relation with the corresponding demand component in unemployment. This assumption is consistent with our "production function"-based interpretations below. Then our procedure correctly distinguishes the dynamic effects of demand and supply components in output and unemployment.

PROOF OF THEOREM: *By* (i)–(vi), *the matrix spectral density of* X *is given by* $S_X(\omega) = B(z)B(z)^*|_{|z|=1}$. *By reasoning analogous to that in* pp. 44–48 *in Rozanov* (1967), *there exists a* 2×2 *matrix function* C, *each of whose elements are analytic, with* $\det C \neq 0$ *for* $|z|\leq 1$, *and* $S_X = CC^*$ *on* $|z|=1$. *This represents* X *as a moving average in unit variance orthogonal white noise, obtainable from its VAR mean square approximation. However, such a moving average* C *need not satisfy the condition that the first (demand) disturbance have only transitory impact on the level of output (condition* (ix)*). Form the* 2×2 *orthogonal matrix* M *whose second column is the transpose of the first row of* C *evaluated at* $z=1$, *normalized to have length* 1, *as a vector. Then* $A \equiv CM$ *provides the moving average representation satisfying* $(viii)$–(x). *For the second part of the theorem, notice that* A *has been constructed so that on* $|z|=1$:

$$|1-z|^2|\alpha_{11}|^2 + |a_{12}|^2$$

$$|1-z|^2\beta'_{11}(\beta'_{11})^*$$

$$+ B'_{12}(B'_{12})^*;$$

$$(1-z)\alpha_{11}a^*_{21} + a_{12}a^*_{22}$$

$$= (1-z)\beta'_{11}(B'_{21})^*$$

$$+ B'_{12}(B'_{22})^*;$$

$$|a_{21}|^2 + |a_{22}|^2 = B'_{21}(B_{21})^*$$

$$+ B'_{22}(B_{22})^*.$$

For e_d *to be orthogonal to* f_s, *and* e_s *to be orthogonal to* f_d *at all leads and lags, it is necessary and sufficient that on* $|z|=1$:

(a) $|\alpha_{11}|^2 = \beta'_{11}(\beta'_{11})^*$;
(b) $|a_{12}|^2 = B'_{12}(B'_{12})^*$;
(c) $\alpha_{11}a^*_{21} = \beta'_{11}(B'_{21})^*$;
(d) $a_{12}a^*_{22} = B'_{12}(B'_{22})^*$;
(e) $|a_{21}|^2 = B'_{21}(B'_{21})^*$;
(f) $|a_{22}|^2 = B'_{22}(B'_{22})^*$.

Consider relations (a), (c), *and* (e). *Denoting complex conjugation of* B *by* $\bar{B}$, *the triangle inequality implies that*:

$$|\beta_{11}\cdot\bar{B}_{21}| = \left|\sum_{j=1}^{p_d}\beta_{11j}B^*_{21j}\right| \leq \sum_{j=1}^{p_d}|\beta_{11j}B^*_{21j}|,$$

where the inequality is strict unless B_{21} *is a complex scalar multiple of* β_{11} *for each* z *on* $|z|=1$. *Next, by the Cauchy-Schwarz inequality*,

$$\sum_j|\beta_{11j}B^*_{21j}| \leq \left(\sum_j|\beta_{11j}|^2\right)^{1/2}\cdot\left(\sum_j|B_{21j}|^2\right)^{1/2},$$

again with strict inequality except when B_{21} *is a complex scalar multiple of* β_{11}, *for each* z *on* $|z|=1$. *Therefore*:

$$|\alpha_{11}a^*_{21}|^2 \leq |\alpha_{11}|^2\cdot|a_{22}|^2, \quad on\ |z|=1,$$

where the inequality is strict except when B_{21} *is a complex scalar multiple of* β_{11} *on* $|z|=1$. *But the strict inequality is a contradiction as*

α_{11} and a_{22} are just scalar functions. Thus (a), (c), and (e) can be simultaneously satisfied if and only if there exists some complex scalar function $\gamma_1(z)$ such that $B_{21} = \gamma_1 \cdot \beta_{11}$. A similar argument applied to (b), (d), and (f) shows that they can hold simultaneously if and only if there exists some complex scalar function $\gamma_2(z)$ such that $B_{22} = \gamma_2 \cdot B_{12}$. This establishes the theorem. □

REFERENCES

Berk, Kenneth N., "Consistent Autoregressive Spectral Estimates," *Annals of Statistics*, May 1974, *2*, no. 3, 489–502.

Beveridge, Stephen and Nelson, Charles, "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'," *Journal of Monetary Economics*, March 1981, 7, 151–74.

Blanchard, Olivier, "A Traditional Interpretation of Macroeconomic Fluctuations," *American Economic Review*, forthcoming, 1989.

______ **and Summers, Lawrence**, "Hysteresis and European Unemployment," in Stanley Fischer, ed., *NBER Macroeconomics Annual*, Cambridge: MIT Press, 1986, 15–78.

______ **and Watson, Mark**, "Are Business Cycles All Alike?," in Robert Gordon, ed., *The American Business Cycle; Continuity and Change*, Chicago: NBER and University of Chicago Press, 1986, 123–56.

Campbell, John Y. and Mankiw, N. Gregory, (1987a) "Are Output Fluctuations Transitory?," *Quarterly Journal of Economics*, November 1987, *102*, 857–80.

______ **and** ______, (1987b) "Permanent and Transitory Components in Macroeconomic Fluctuations," *American Economic Review Proceedings*, May 1987, 111–17.

Christiano, Lawrence J., "Searching for a Break in GNP," NBER Working Paper No. 2695, August 1988.

Clark, Peter, "The Cyclical Component in U.S. Economic Activity," *Quarterly Journal of Economics*, November 1987, 797–814.

______, "Trend Reversion in Real Output and Unemployment," Stanford University, mimeo., 1988.

Cochrane, John, "How Big Is the Random Walk in GNP?," *Journal of Political Economy*, October 1988, *96*, no. 5, 893–920.

Diebold, Francis and Rudebusch, Glenn, "Long Memory and Persistence in Aggregate Output," Finance and Economics Discussion Series, Division of Research and Statistics, Federal Reserve Board, January 1988.

Evans, George, "Output and Unemployment Dynamics in the United States: 1950–1985," London School of Economics, 1987.

Fischer, Stanley, "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy*, February 1977, *85*, no. 1, 191–205.

Gali, Jordi, "How Well Does the AD/AS Model Fit Postwar U.S. Data?," MIT, July 1988.

Granger, Clive and Morris, M. J., "Time Series Modeling and Interpretation," *Journal of the Royal Statistical Society*, 1976, *139*, Part 2, 246–57.

Harvey, Andrew, "Trends and Cycles in Macroeconomic Time Series," *Journal of Business and Economic Statistics*, July 1985, *3*, no. 3, 216–27.

Jun, Sung-in, "Neutrality of Money, Wage Rigidity, and Co-Integration," MIT, May 1988.

Nelson, Charles and Plosser, Charles, "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics*, September 1982, *10*, 139–62.

Perron, Pierre, "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis," Cahier de recherche no. 3887, CRDE, Universeté de Montréal, 1987.

Prescott, Edward, "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, 9–22.

Quah, Danny "The Relative Importance of Permanent and Transitory Components: Identification and Some Theoretical Bounds," MIT Working Paper No. 498, October 1988.

Rozanov, Yu A., *Stationary Random Processes*, San Francisco: Holden-Day, 1967.

Shapiro, Matthew and Watson, Mark, "Sources of Business Cycle Fluctuations," in Stanley Fischer, ed., *NBER Macroeconomics Annual*, Cambridge: MIT Press, 1988, 111–48.

Sims, Christopher, "The Role of Approximate Prior Restrictions in Distributed Lag Estimation," *Journal of the American Statistical Association*, March 1972, *67*, no. 337, 169–75.

Taylor, John B., "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, February 1980, *88*, 1–24.

Watson, Mark, "Univariate Detrending Methods with Stochastic Trends," *Journal of Monetary Economics*, July 1986, *18*, 49–75.

U.S. Department of Labor, Bureau of Labor Statistics, *Labor Force Statistics Derived from the Current Population Survey*, Vol. 2, September 1982.

——, *Employment and Earnings*, February issues.